MathSnacks

Poincaré and Perelman: Perfection!

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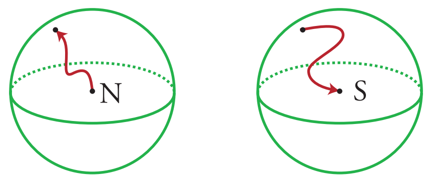
Dunkin' and Donuts

The famous cliché is that to a *topologist* a coffee cup is the same as a donut. This is because one can be deformed into the other without cutting or tearing. By contrast, it is impossible to deform a donut into a sphere.

Spherical Simplicity

How can we tell? The sphere S^2 has the *Simple Loop Property*: any loop on the sphere can be contracted like a rubber band to the North Pole. By contrast, on any other *closed* surface, such as a donut, we can tie a loop which *cannot* be shrunk to a point.

Disturbing Dimension

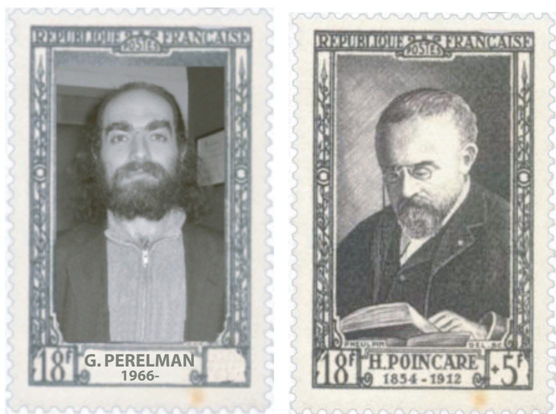


It is difficult to imagine *any* 3D-world, apart from the Euclidean World (which is not bounded and so not closed). Here are two ways to think of the 3D-sphere, S^3. Firstly, we can think of S^3 as all points (x,y,z,w) in 4D-space which satisfy the equation

$$x^2+y^2+z^2+w^2=1.$$

Secondly, we can make an *atlas* of S^3, consisting of two distorted *maps,* one of the Northern Hemiball, and one of the Southern Hemiball; this is exactly like making an atlas for the Earth, with one map-disk for each Hemisphere, and the Equator corresponding to the edges of the map-disks. Note that the North and the South Poles of S^3 correspond to the centres of the two map-balls and the Equator corresponds to the surfaces of the two map-balls. If we start in one Hemiball and go cross the Equator, we'll appear in the other Hemiball.

Perelman Proves Poincaré



In 1904, Henri Poincaré made his famous Conjecture, that S^3 was the only (closed) 3D-world with the Simple Loop Property. 100 years later, based upon the work of Richard Hamilton, Gregory Perelman proved the Poincaré Conjecture. Perleman was awarded (and declined) the 2006 Fields Medal.

Ripper Reference*

Jeffrey Weeks, The Shape of Space, CRC, 2001
Sylvia Nasar and David Gruber, Manifold Destiny, www.newyorker.com/fact/content/articles/060828fa_fact2