MathSnacks

To Be or Not to Be
Four Variations on Mathematical Existence

by Marty Ross, Burkard Polster, and QED (the cat)

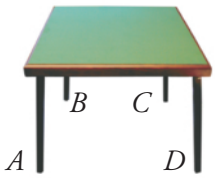
Hairy Twin*

Somewhere in the city of Melbourne there are two people with exactly the same number of hairs.

The following is a lovely application of the so-called *Pigeonhole Principle*.

Proof: We know that nobody has more than 200,000 hairs on their heads. Imagine this number of houses, labelled from 1 to 200,000, and then ask every Melbournian to move into the house labelled by the number of their hairs. Since there are more Melbournians than houses, there must be a house with at least two people in it. And, of course, any two such people in the same house have the same number of hairs.

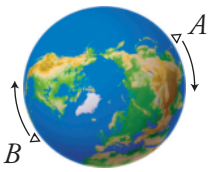
Table Turning



Put a square table on an irregular surface and chances are that it will wobble. However, by just turning it on the spot, you can always find a position in which all four legs touch the ground.

Proof: Suppose legs A, B, and C are touching the ground, and leg D is hovering in the air. So, if we anchor B and C, and force D to touch the ground, then A would be forced into the ground: bad for the table! Now rotate the table 90° clockwise, ensuring that A, B, and C are always touching the ground. Then, we again end up in a bad situation, as now D (which has assumed A's position) is poking into the ground. Since D starts out above the ground, and ends up below, there must be an intermediate position where D, and therefore all four legs, are touching the ground.

Mathematical Metereology*

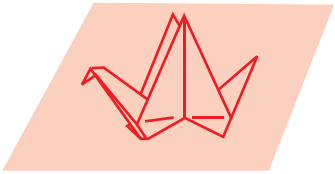


At any point in time there are two antipodal points on the equator at which the temperatures coincide.

Proof: Start by comparing opposite points A and B on the equator. If A is warmer than B, then we simultaneously rotate both points around the equator until they have swapped places. But now A is cooler than B. Therefore, since the temperature along the equator varies *continuously*, somewhere along the way the two opposite points must have the same temperature.

If we are willing to leave the equator, we can say more: there are antipodal points on the Earth where both the temperature *and* the air pressure coincide! However, this is much harder to prove.

Beautiful Points*



Lay a square sheet of paper flat on the ground. Take an identical sheet of paper, turn it into an origami sculpture and place it on top of the first sheet. Then there must be a point on the second sheet which lies directly above the corresponding point on the first sheet.

This is the famous and difficult *Brouwer Fixed Point Theorem*. It is at the heart of the mathematican John Nash's Nobel Prize-winning work on Game Theory. It is also how, in the movie *A Beautiful Mind*, Nash determines how he and his colleagues can best compete for the attention of their pretty companions.

Ripper Reference*

Burger & Starbird, The Heart of Mathematics, Springer, 2004.