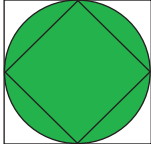
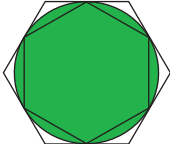


MathSnacks by Marty Ross,
how i wish i could Burkard Polster,
calculate pi and QED (the cat)

Piblematic



$\pi = 3$

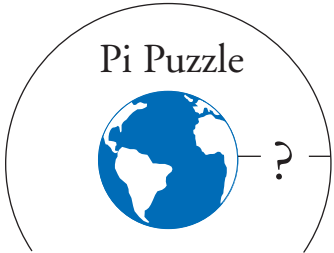


Well, so implies the Bible: *And he made a molten sea (vessel), ten cubits from the one brim to the other: it was round ... and a line of thirty cubits did compass it round about.* (I Kings 7:23). Other ancient approximations for π are $3 \frac{1}{8}$ (Mesopotamia, c. 1800 B.C.) and $(16/9)^2$ (Egypt, c. 1650 B.C.).

What π really is is the ratio of the *circumference* of any circle to its *diameter*. Archimedes (c. 250 B.C.) was the first to use this definition to properly estimate π . By inscribing and circumscribing a circle with regular polygons, he proved that π lies between $3 \frac{10}{71}$ and $3 \frac{1}{7}$. (What are the approximations given by the squares and hexagons above?) More importantly, Archimedes' method can be used to obtain π to any desired accuracy.

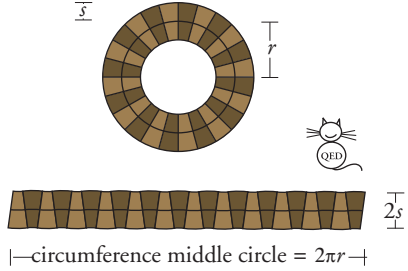
Of course, all of the above rational values for π (including the decimal number hidden in our title) can only be approximations, since π is irrational (Joseph Lambert, 1766).*

Pi Puzzle



A simple formula can give unexpected answers! Suppose there is a loop of string that is tightly wrapped around the Equator. After lengthening the loop by one meter, it will lie a bit above the surface of the Earth: how far? Compare your result to the radius of a circle whose circumference is one meter. What happens if you replace the Earth by a sphere of different radius, but leave the rest of the thought experiment unchanged.

Pi Pies



—circumference middle circle = $2\pi r$ —

From the formula for the circumference of a circle, a simple idea allows us to derive the area formulae for rings and circles. Take a circle of radius r and thicken it to make a ring of width $2s$. Dissect the ring and then arrange the resulting slices into a wobbly rectangle. As we use more and more slices, we obtain *exactly* a rectangle. Hence

$$\text{area ring} = \text{area rectangle} = 4\pi rs.$$

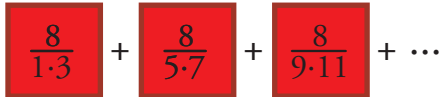
By choosing $r = s$, the ring turns into a circle of radius $R = 2r$. This gives the area formula for a circle:

$$\text{area circle} = \pi R^2.$$

Now, think of the first diagram as the view from above of a do-nut, a *torus*. Dissecting and rearranging as before, eventually gives a cylinder, (which from the top looks like a rectangle). Hence

$$\begin{aligned} \text{surface area torus} &= \text{surface area cylinder} = 2\pi r \cdot 2\pi s, \\ \text{volume torus} &= \text{volume cylinder} = 2\pi r \cdot \pi s^2. \end{aligned}$$

Precisely Pi



This infinite sum is equivalent to the oldest explicit expression for π (Madhava, c. 1400). There are many, many others, the flipside of which is that π make surprising appearances in the solutions to many problems. For example, pick two natural numbers m and n at random. What is the probability that m and n have no common factors? The answer is $6/\pi^2$!

Ripper References*

P. Beckmann, *A History of Pi*, St.Martin's Press, 1971.
Pi (the movie, 1998), available on VHS and DVD.