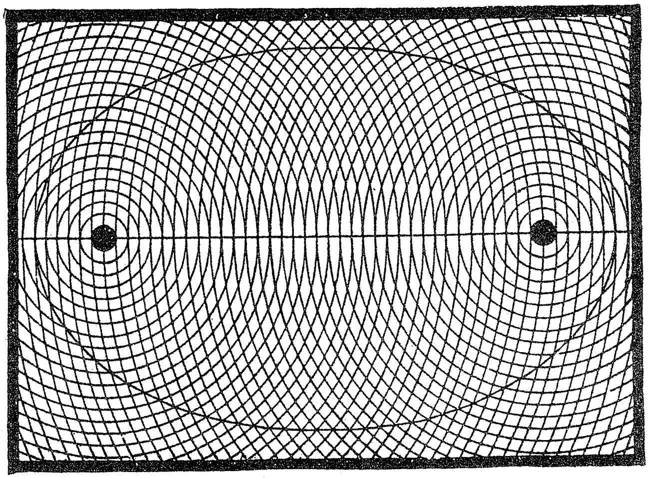
Function

A School Mathematics Journal

Volume 26 Part 3 June 2002



School of Mathematical Sciences – Monash University

Reg. by Aust. Post Publ. No. PP338685/0015

Function is a refereed mathematics journal produced by the School of Mathematical Sciences at Monash University. Founded in 1977 by Professor G B Preston, *Function* is addressed principally to students in the upper years of secondary schools, but more generally to anyone who is interested in mathematics.

Function deals with mathematics in all its aspects: pure mathematics, statistics, mathematics in computing, applications of mathematics to the natural and social sciences, history of mathematics, mathematical games, careers in mathematics, and mathematics in society. The items that appear in each issue of *Function* include articles on a broad range of mathematical topics, news items on recent mathematical advances, book reviews, problems, letters, anecdotes and cartoons.

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Function is published five times a year, appearing in February, April, June, August, and October. Price for five issues (including postage): $27.50* ; single issues $7.50. Payments should be sent to: Business Manager, *Function*, School of Mathematical Sciences, PO Box 28M, Monash University VIC 3800, AUSTRALIA; cheques and money orders should be made payable to Monash University. For more information about *Function* see the journal home page at

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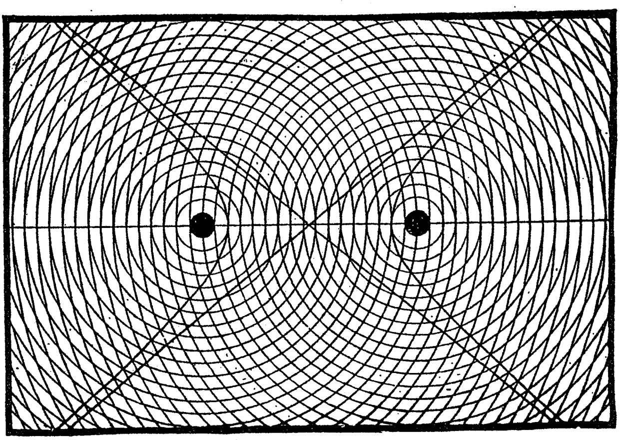
* $14 for *bona fide* secondary or tertiary students.

THE FRONT COVER

Our front cover for this issue comes from *Scientific American Supplement*, August 7, 1886. They in their turn reprinted it from a journal called *Knowledge*. It illustrates an article called "Drawing Ellipses", and it depends on a property of the ellipse that has long been used in making diagrams of them. Place two points O_1, O_2 a distance $2c$ apart, and let P be a point on the ellipse. The points O_1, O_2 are termed the *foci*. Let $O_1P = r_1$, $O_2P = r_2$ and $a > c$; draw the set or locus of all points P such that $r_1 + r_2 = 2a$. This set of points constitutes the ellipse.

A once popular way to draw an ellipse took the following form. Drawing pins were pushed into a sheet of paper at points corresponding to O_1, O_2. A string of length $2a$ was held by these pins at its two ends, and then stretched taut by a pencil which was then constrained to move only on the ellipse, whose points were thus recorded by the trace of the pencil on the paper.

Our cover diagram demonstrates another approach. Around each of the foci, supposed 50 units apart, circles (in the example, 58 in number) are constructed at regular unit intervals. The foci themselves are highlighted by blacking in the innermost one of each of these two sets of circles. The ellipse is constructed in the figure by considering the locus of all points such that $r_1 + r_2 = 64$. The same principle may be used to construct a hyperbola (see below), except that its two branches are given by $r_1 - r_2 = \pm 2a$. This same construction was used in the cover illustration for *Function*, *Vol* **17**, *Part* **5**, for which a physical interpretation was given.



THE BOUNDS OF THE UNIVERSE

A BRIEF HISTORY OF ASTRONOMY: PART 1

K C Westfold

[This is the second in our series of astronomical expositions by the late Professor Westfold. Eds]

The study of Astronomy dates from the earliest times, some thousands of years BC. In those days, and especially in the Middle Eastern lands which are the cradle of civilization, people could not help spending much of their time contemplating the passage of the heavens across the celestial sphere. Moreover, as their civilization developed, they would need some means of direction-finding for travel on land and sea.

They would note the regular appearance of the Sun, rising in the East and setting in the West. The daylight hours were immediately preceded and succeeded by twilight. In the darkness the stars became visible, and their regular patterns of motion across the sky would be noted.

The varying altitude of the Sun at midday throughout the year was correlated with the succession of the seasons, which affected both hunting and agriculture. As early as 2000 BC, the Babylonians had recognized a year of 360 days. Meanwhile the varying phases of the moon had given them the period of the lunar month. There were 12 such months in the Babylonian year and this division necessitated interpolation of an extra month from time to time in order to square with the seasons.

It would soon be noted that the Sun in one year moved in a closed path relative to the stars and in a direction opposite to the daily motions of the stars, which was East to West. The Moon too had an annual motion similar to that of the Sun, and attention was directed to a band around the Sun's path, now called the ecliptic. This band of stars was called the zodiac, and by 700 BC or even earlier it was divided into 12 "signs" or constellations corresponding to where the Sun happened to lie in each month of the year.

It would then be observed that the Moon and certain brighter objects in the zodiac, while having a general motion to the East like the Sun, wandered to and fro across the ecliptic with varying periods. These are the *planets* named from the Greek word for "wanderer". The rapidity of their motions was taken as a measure of distance; the more slowly moving Saturn, Jupiter and Mars were considered to be farther from the Sun, and were dubbed "superior" planets, while the faster Venus and Mercury were nearer and were called "inferior". The Moon was regarded as closer than these because of the occurrence of "occultation" when the Moon passes between a planet or star and ourselves.

The interest of the Egyptians and the Mesopotamians in Astronomy was primarily utilitarian: for such purposes as direction-finding, telling the onset of the seasons and predicting eclipses (regarded as omens). In the 5th and 6th centuries BC, there arose the Ionian and Greek Mathematical Astronomy which was pursued for its own sake. It was the *Almagest* of Ptolemy and other Greek writings (preserved by the Mohammedan conquerors of Alexandria) which provided the foundation for the new progress in Astronomy that came with the Renaissance.

By the time of *Aristotle* (384-322 BC), the Greeks imagined all the stars fixed to a finite sphere with the earth as its centre. This rotated uniformly once a day. Within this sphere were other spheres on which the Sun, Moon and planets were fixed. These rotated uniformly, and complex arrangements were necessary to account for the observed motions.

Aristarchus of Samos (310-230 BC) anticipated the Copernican view that the Sun lay at the centre of the sphere of fixed stars and that the earth rotates on an axis and revolves about the Sun. He invented an ingenious method for measuring the distance of the Sun relative to the distance of the Moon.

Eratosthenes of Cyrene (276-196 BC) measured the radius of the earth by determining the angle the Sun's rays made with the vertical at Alexandria when it was vertically overhead at Cyrene 5000 stadia further south.

But the greatest of the Greek astronomers was *Hipparchus of Nicaea* (190-120 BC). He can be regarded as the founder of systematic observational Astronomy, collecting and collating records of past and contemporary observations. By comparing his records with those made 150 years earlier he discovered the phenomenon of the precession of the equinoxes in a westerly direction. [The equinoxes are the intersections of the ecliptic with the equator of the celestial sphere.] His early observations were so accurate that his successors were able to predict the times of eclipses of the Sun and the Moon to within a few hours.

Ptolemy (c140 AD) worked in the flourishing centre of Alexandria. He wrote the *Almagest*, whose importance has already been noted. it is based on earlier observations, mainly those of Hipparchus. To account for the apparent motions of the heavenly bodies, he adopted a system centred at the earth, and with each planet revolving uniformly in a circle (the "epicycle") whose centre revolved uniformly in a circle (the "deferent") centred at some distance from the earth. There are sufficient parameters to adjust in this model that the system accords quite well with observations made by the crude instruments of the time.

He also obtained the distance to the moon in terms of the radius of the earth, by means of the method still in use today: that of *parallax*. This uses bearings taken from two different places on the surface of the earth, giving the angles between the line of sight to the object and the local vertical. The angle between these bearings and the distance between the observation posts determine the distance to the object. This distance determination is the yardstick which enables the furthest distances of other members of the solar system to be measured. Ptolemy also noted the phenomenon of refraction of light by which the apparent altitude of stars is increased.

The school of Astronomy in Alexandria remained moribund between the death of Ptolemy and the capture of the city by the Arabs in 640 AD. In the next century the Arabs themselves became interested in Astronomy, and although they did not produce any original theories they took careful observations and simplified the computations of Astronomy with their enormously simpler arithmetic notation. Their greatest service is perhaps that they kept astronomical knowledge alive during the dark ages.

The Renaissance was characterized by a new spirit of enquiry with the tendency to question all aspects of the Aristotelian scheme of Physics, which had by this time (and contrary to the tenets of Aristotle himself) achieved the status of dogma. The new spirit subjected old beliefs to the test of experience. Among the dogmas so questioned was the Ptolemaic astronomical system.

Copernicus (1473-1543) saw that the motions of the stars could equally well be accounted for by the hypothesis of the daily rotation of the earth about an axis as by the rotation of a stellar sphere. The difficulty that Ptolemy had had with this notion was that he thought that bodies on the earth's surface would be swept off if the earth itself rotated. However, if the earth's atmosphere also rotated with the earth, this difficulty could be overcome. Copernicus took the further step of supposing the earth and the planets to revolve in orbits around the Sun. Unfortunately however he was still tied to the Greek concept that circular motions were the appropriate paths of "celestial bodies". Thus he was forced to adopt a system of epicycles and deferents, so that his system was little better than the Ptolemaic. For such a geometrical description, no dynamical theory is conceivable.

The way to a simpler geometrical description was opened by the remarkably accurate observations of *Tycho Brahe* (1546-1601), made with instruments he designed and built himself. Despite the extreme accuracy of his observations, he could detect no parallax in the positions of the stars. That is to say, they appeared not to move at all relative to one another, as one might expect if they were seen from a moving earth. This could have been explained if the stars were so far away that their apparent motions were imperceptible. Their distance would have to be large in relation to the distance from the earth to the Sun. Brahe, believing this conclusion to be in conflict with Christian teaching, thus rejected the Copernican system and proposed another. He had the planets revolving about the Sun, which itself revolved about a fixed earth.

Associated with Brahe while he was compiling his observations of the planets and cataloguing the stars was *Kepler* (1571-1630), who set himself the task of computing the planetary orbits in space from Tycho's observations. He found both Ptolemy's and Brahe's systems defective when compared with the observations. After an incredible amount of computation, he arrived at his three laws of planetary motion.

These were:

1. The orbit of each planet is an ellipse with the Sun at one of its two foci.
2. The line joining the Sun to any planet sweeps out equal areas in equal times.
3. The squares of the periods of revolution of the planets are proportional to the cubes of their distances from the Sun.

These laws enabled *Newton* to make the dynamical inferences that led to his *Law of Universal Gravitation*.

Galileo (1564-1642) should be chiefly noted for his successful pursuit of the scientific method of discovery. It is difficult for us to realize today the magnitude of his achievement in emancipating himself from the *a priori* Aristotelian method of reasoning (speculating in the absence of hard data). He was the first to show that a heavy body falls no faster than a lighter one; in doing so he laid the foundations of the experimental science of Mechanics. His brilliant advocacy of the Copernican theory, supported by observations with his telescope, brought about his clash with the ecclesiastical authorities who represented the old learning. Among the other discoveries he made with the use of his telescope, we may note the resolution of the Milky Way into stars similar to those found in other parts of the sky.

Newton (1642-1727) was perhaps the greatest scientific genius of all time. He discovered the binomial theorem in Mathematics and invented the calculus; he discovered the diffusion of light by a prism. (This has since become one of the astronomer's principal tools in determining the composition of stellar material.) He invented the reflecting telescope which overcame technical difficulties with the earlier refracting telescopes of Galileo and others.

But his greatest achievement was the Law of Universal Gravitation, which he deduced from Kepler's Laws. He was able to demonstrate that the second law implied an acceleration of the planet towards the Sun, and that the first law implies that the acceleration is inversely proportional to the square of the distance between the two. The third law he showed proved that the attractive force of the Sun was in proportion to the mass of the planet. His laws of motion formally comprise the basic principles of Mechanics, still of widespread application today.

He was able to verify his principles by means of an analysis of the Moon's motion, the flattening of the rotating earth, and the behaviour of the tides. He also showed that the attraction of the Moon (and to a lesser degree, the Sun) explained the precession of the equinoxes. He was able to calculate and compare with observation the extent of this phenomenon. He calculated that the point where the ecliptic crosses the celestial equator moved westward at about 50 seconds of arc per year. This remains the accepted figure.

Newton's discoveries founded the science of Celestial Mechanics. Later names are *Euler* (1707-1783), *Clairaut* (1713-1765), *d'Alembert* (1717-1783), *Lagrange* (1736-1813) and *Laplace* (1749-1827). Even today, there is still research in this area to elucidate the complex motions of planets, comets and satellites in the solar system. It has also extended its scope to take in the Milky Way and beyond.

The most significant discoveries of this time were those of *Römer* (1644-1710), who measured the velocity of light in 1675, and *Bradley* (1693-1762), whom we credit with the theory of planetary nutation and work on the aberration of light.

By observing eclipses of the moons of Jupiter, Römer found that in opposition, when the earth is closest to Jupiter, the eclipse occurred earlier than expected, whereas in conjunction, when the earth and Jupiter are furthest apart, it was later. He explained this by allowing for the extra time the light took to travel the extra distance. This enabled him to calculate the velocity of light.

Bradley found that the star γ-Draconis, in the course of a year, apparently moved in a small ellipse on the celestial sphere. The "obvious" explanation would attribute this effect to the parallax due to the earth's motion, but Bradley showed that the orientation of the ellipse put this explanation out of court. Rather, the effect is due to the combined velocities of the incoming light and of the earth. The explanation came to him when he noticed the change in direction of a boat's pennant when the boat changed course in a steady wind.

The compilation of accurate star positions necessitated the reduction of effects due to aberration and parallax, and also those caused by the precession of the equinoxes. Even after allowing for all these effects, Bradley still found discrepancies between observations made in different years. He found an underlying periodicity of $18\frac{1}{2}$ years. He attributed this to a "wobble" of the earth's axis, superposed on the precession. This "wobble" is the nutation.

Both precession and nutation can be understood in terms of Newtonian mechanics.

[To be concluded in our next issue.]

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THE POWER TRIANGLE AND SUMS OF POWERS

Jim Cleary, Mount Lilydale Mercy College

Function readers will all be familiar with the formula

$$1+2+3+\ldots+m=\frac{m(m+1)}{2} \tag{1}$$

and may also have seen a related one

$$1^2+2^2+3^2+\ldots+m^2=\frac{m(m+1)(2m+1)}{6} \tag{2}$$

If we denote by $C(m,r)$ the number of different ways in which a subset of r items may be chosen from a set of m items, then $C(m,r)=\dfrac{m!}{r!(m-r)!}=\dfrac{m(m-1)(m-2)\ldots(m-r+1)}{1\times2\times3\times\ldots\times r}$. This expression is sometimes denoted by other symbols such as $\begin{pmatrix} m \\ r \end{pmatrix}$ or ${}^mC_r$ but here I will write $C(m,r)$. The numbers $C(m,r)$ are referred to as "binomial coefficients". It may now readily be checked that the right-hand side of Equation (1) is equal to $C(m,1)+C(m,2)$.

But could we not also write the more general expression

$$1^p+2^p+3^p+\ldots\ldots+m^p,$$

where p is a positive integer, also as a sum of binomial coefficients?

To explore this question, begin with $p = 2$, and try

$$1^2+2^2+3^2+\ldots\ldots+m^2=aC(m,1)+bC(m,2)+cC(m,3)$$

We may find values for a, b, c by putting low values of m into the proposed equation. Start with $m = 1$, and note that $C(1,2)=C(1,3)=0$, because there is *no* way of selecting a subset of 2 or 3 items from an initial set containing only 1. This tells us that $a = 1$. From the next case

$m = 2$, we deduce that $b = 3$, and finally from the case $m = 3$, we deduce that $c = 2$.

This leads us to suggest that

$$1^2 + 2^2 + 3^2 + \ldots.. + m^2 = C(m,1) + 3C(m,2) + 2C(m,3).$$

The right-hand side of this expression simplifies to $\dfrac{m(m+1)(2m+1)}{6}$ which is indeed the correct formula, from Equation (2).

Applying the same approach to the next case, $p = 3$, results in the formula

$$1^3 + 2^3 + 3^3 + \ldots.. + m^3 = C(m,1) + 7C(m,2) + 12C(m,3) + 6C(m,4)$$

and this time the right-hand side simplifies to $\left(\dfrac{m(m+1)}{2}\right)^2$ which again is the correct answer.[1]

If we can find the pattern behind the coefficients, we will produce a simple means of calculating the sum $1^p + 2^p + 3^p + \ldots.. + m^p$. The coefficients discovered so far may be placed in a triangular pattern:

$$\begin{array}{ccccccc} & & 1 & & 1 & & \\ & 1 & & 3 & & 2 & \\ 1 & & 7 & & 12 & & 6 \end{array}$$

Observe that each row begins with a 1 and that the nth row ends with $n!$ We could even complete the triangular pattern by placing a single 1 at the apex, as mathematicians usually define 0! to be 1.

To obtain the other elements, label the terms as shown by the small bracketed numbers in the display opposite. Multiply each of two adjacent entries in a selected row by their respective labels and add the results to get an entry in the next row, much as in the case of Pascal's Triangle. I call the resulting pattern "the Power Triangle". Here is how it works.

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[1] See *Function*, April 2001, p. 58. Eds.

$$1_{(1)}$$
$$1_{(1)} \quad 1_{(2)}$$
$$1_{(1)} \quad 3_{(2)} \quad 2_{(3)}$$
$$1_{(1)} \quad 7_{(2)} \quad 12_{(3)} \quad 6_{(4)}$$
$$1_{(1)} \quad 15_{(2)} \quad 50_{(3)} \quad 60_{(4)} \quad 24_{(5)}$$
$$1_{(1)} \quad 31_{(2)} \quad 180_{(3)} \quad 390_{(4)} \quad 360_{(5)} \quad 120_{(6)}$$

The labels form an obvious pattern, and the actual entries are readily calculated. For example, the third entry in the fifth row (180) comes from the second and the third entries in the fourth row:

$$15 \times 2 + 50 \times 3 = 180.$$

Entries at the ends of the rows may also be seen to follow the same pattern if the rows are thought of as extending to both right and left, but with all the other entries as zero.

We may check the correctness of our formula by looking at the next case, $p = 4$. We have

$$1^4 + 2^4 + \ldots + m^4 = C(m,1) + 15C(m,2) + 50C(m,3) + 60C(m,4) + 24C(m,5)$$

where the coefficients come from the fourth row of the Power Triangle.

If we simplify the right-hand side of this expression, we reach the expression $\dfrac{m(m+1)(2m+1)(3m^2+3m-1)}{30}$ and once again, this is the correct answer.

Similar results apply for higher values of p.

[We believe that Mr Cleary's approach is a new one. It seems simpler than other methods for calculating these sums. It is possible to prove that the method works universally, although the proof is a little difficult for *Function*. However, the different formulae derived for the individual values of p are each capable of proof by the method of mathematical induction. Eds][2]

[2] These formulae (up to $p = 10$) may be found at http://m216.net/math/math/math/f/f050.htm. The reader may use these results to check the accuracy of the formulae given by the power triangle. Eds

HISTORY OF MATHEMATICS

Queensland's Mathematical Judge

Michael A B Deakin

The colony of Queensland separated from New South Wales in late 1859, and set about establishing for itself those aspects of civil order that such a newly founded jurisdiction would need. The sole resident judge in the new colony was A J P Lutwyche, who applied for the newly vacant post of Chief Justice. However, he had fallen out with the Government of the day, as also with the Governor, and the post was not awarded to him, but rather to an English barrister named James Cockle.

James Cockle had been born in 1819, the second son of an English surgeon. On completion of his regular schooling, just short of his eighteenth birthday, he went abroad and spent a year in the West Indies and the United States. On his return to England, he entered Trinity College, Cambridge. This was in 1837, and his area of interest was Mathematics. He sat the final examination in 1841.

The examination in Mathematics (a difficult one) was known as the Tripos, and those who achieved first class honours were entitled to be called Wranglers. The Wranglers were graded in order of merit, from the First, or Senior Wrangler, on down through the Second, Third and so on. It was by no means always the case that the best students came out on top. Hardy, of whom I wrote in June 1995, was only Fourth Wrangler in his year. However in 1841, the Senior Wrangler was Stokes, who went on to achieve great prominence in Applied Mathematics. (The basic equations of Fluid Flow, the Navier-Stokes Equations, preserve his name.)

The young James Cockle, however, was left a long way behind. He ended up in the 33rd place, and so cut short any aspirations he might have had towards a career in Mathematics. He had already shown an interest in the Law, and so this other interest came to take precedence as the foundation of his career. He took degrees in 1842 and 1845, and practised as a special pleader from 1845 till 1849. He was called to the

Bar in 1846, and continued to practise as a barrister until his move to Queensland in 1863.

Although Lutwyche had been passed over for a younger man, and one moreover without any previous experience as a judge, he and Cockle became close friends, and remained so in both personal and professional capacities until Cockle returned to England in 1879 and Lutwyche died in 1880. Cockle was knighted in 1869, and even after his return to England, he continued to take an interest in colonial affairs.

However, by then, he had come into a position where his earlier interest in Mathematics could be pursued as a full-time occupation. He had begun writing mathematical papers as early as 1841, and had some 50+ to his credit when he took up the Queensland position. Another 40 or so were sent to British journals from Brisbane. It was early in this period (1865) that he was elected a Fellow of the Royal Society. It was then also that he instituted and endowed the Cockle Prize in Mathematics, that is still awarded at Brisbane Grammar School to their top student in Mathematics.

He retired from the bench on a handsome pension that allowed him to pursue his mathematical interests, and his research continued. The years of his retirement saw another 30+ papers, and also a flurry of organisational work in the service of Mathematics. He was active in the running of several learned societies, most particularly the London Mathematical Society, whose president he became in 1886 and again in 1887.

His health may by then have been beginning to fail, for his widow later recalled: "So determined was he to be present at his installation as President, that, although suffering from a bad attack of congestion of the lungs, he had himself wrapped in blankets, which were only removed on his arrival." Certainly, by 1892, he was only seldom able to attend meetings, whereas he had previously made a point of being there. He died in 1895.

At the time of his stay in Brisbane, there were only three mathematicians, properly so called, in Australia. The Professor in Sydney was M B Pell, an actuary, and his Melbourne counterpart was W P Wilson, an astronomer. The third was Cockle. Of these three, Cockle would have had the best reputation as a mathematician back in England. Not only did he publish more prolifically in mathematical journals there, but his focus of interest was more solidly pure mathematical.

He studied two major areas of interest in his research. The first was the Theory of Equations, by which is meant the body of theory relevant to the solution and properties of polynomial equations; the other was the study of differential equations. These latter are equations in which an unknown function $y(x)$ is to be found from information concerning one or more of its derivatives. He is also credited with the discovery of a connection between these two areas of interest.

At this time, the challenge was to solve the quintic, the equation of 5th order. Linear (first order) and quadratic (second order) equations had long been routinely solved. Cubic (third order) and quartic (fourth order) equations began to be solved in the late Middle Ages, and the theory of both was relatively complete by the middle of the 17th century. The next member of the hierarchy proved much more difficult.

What is at issue is the ability to produce a "formula" that gives the roots in terms of the coefficients and the simple operations of Arithmetic: Addition, Subtraction, Multiplication, Division, Raising Numbers to Powers and the Extraction of Roots. Students will be familiar with the formula for the roots of the quadratic $ax^2 + bx + c = 0$, where the coefficients are a, b and c. According to the formula, the roots are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

It will be seen that an explicit formula is produced and furthermore that no more than the simple operations of Arithmetic are involved.

It is now known (mostly from the work of Abel in the first half of the 19th century) that the goal of solving the general quintic in such a way is unachievable. However, this was not a result that was readily accepted. Cockle's friend Harley wrote of him: "for many years his labours .... were inspired by the hope of 'solving the quintic,' or to be more exact, expressing a root of the general equation of fifth degree by a finite combination of radicals and rational functions. ..... . Cockle considered [Abel's] argument with care and reproduced it as modified by Sir W. R. Hamilton .... [however, he] for many years clung to the conviction that what had been done for the lower equations might be done also for the equation of the fifth degree."

Only one of Cockle's results in this area has stood the test of time. He was able to simplify one line of research in this area to produce clear improvements on the best previous work. This result provided the impetus for a further improvement by Sir Arthur Cayley, a contemporary of Cockle's and a better-known mathematician today.

In the area of differential equations, also, only one of Cockle's results made an impact. This was a technical result that briefly stood as the best available treatment until it was superseded by a more general theorem by the French mathematician Laguerre.

The work connecting polynomial equations with differential equations was inspired by the desire the better to understand the properties of the quintic. It may be described as "ahead of its time" for it is extremely difficult to implement without a computer algebra package. Now that these are available, there has been a revival of interest, and the MATHEMATICA poster on the quintic makes use of it. In hard form, this is displayed at both Monash University and the University of Melbourne, or readers may care to consult it online at

http://library.wolfram.com/examples/quintic/main.html

To get an idea of what is involved, consider the quadratic equation $ax^2 + bx + c = 0$, and reduce it to the form

$$X^2 + 2X + A = 0,$$

where $X = \dfrac{2ax}{b}$ and $A = \dfrac{4ac}{b^2}$. Clearly if we can solve the simpler equation, then we can also solve the more general one; a similar principle applies to the cubic, quartic and quintic equations.

Cockle demonstrated that the solution to the simplified form of the quadratic could be solved by means of the differential equation

$$2(1-A)\frac{dX}{dA} + X = 1.$$

Readers may care to investigate this as an exercise.

At the end of the day, we must say that Cockle was at best a very minor mathematician. Even his friend Harley, who wrote his obituary, conceded: "Of his [numerous] papers given to the mathematical world, many are no doubt slight and fragmentary." Looking at them today, one feels bound to concur.

When a person achieves eminence in two very different fields, as Cockle did, it is difficult to make a balanced assessment of his life. In the (UK) *Dictionary of National Biography*, Cockle is given a place in the Supplement. The author of that article, coming from a legal background, adjudges Cockle's mathematical work as being of more significance than his legal career. However, a new edition of this work is due to appear in 2004, and I have had access to a draft of its article on Cockle. That draft is written by a mathematician, and it takes the opposite view!

COMPUTERS AND COMPUTING

SCIENTIFIC PROGRAMMING

J C Lattanzio, Monash University

There is still a lot of truth to the old advice about learning programming: you must simply start writing programs and gradually increase their complexity. This paper should help you get started.

Suppose a teacher wants to keep a class quiet for an extended period. He/she may ask the students to add all the integers between 1 and 100. An *algorithm* to do this (for a more general case, n replacing 100) is

```
READ n
SET sum TO 0
FOR i = 1 TO n DO
        REPLACE sum BY sum + i
WRITE n, sum
```

With a little thought it is possible to reduce the amount of work done by devising a new algorithm. Legend has it that Gauss (when he was seven years old!) summed the integers from 1 to 100 very quickly in his head by using the following "trick":

$$\begin{array}{llllllll} & 1 & +2 & +3 & \ldots & +98 & +99 & +100 \\ + & 100 & +99 & +98 & \ldots & +3 & +2 & +1 \\ = & 101 & +101 & +101 & \ldots & +101 & +101 & +101 \end{array}$$

Gauss' trick exploits the symmetry:

$$2(1 + 2 + 3 + \ldots + 99 + 100) = 100\times101$$

Therefore

$$1 + 2 + 3 + \ldots + 99 + 100 = 100\times101/2=5050.$$

A simple generalisation yields

$$1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}.$$

Thus a simpler algorithm to achieve the same result would be

```
READ n
SET sum TO n(n + 1)/2
WRITE n, sum
```

Now suppose we want to calculate the reciprocal of any given number x. A suitable algorithm would be:

```
READ x
IF x ≠ 0 THEN
    SET y = 1/x
    WRITE x, y
ELSE
    WRITE "ERROR"
```

We can convert this very simple program into a standard programming language like Pascal or Fortran quite easily.

[The choice of which language to use for your programming always causes arguments. Like the arguments about operating systems

(Mac, Windows, Linux or other Unix, etc) it really depends on the purpose. Your initial programming experience is likely to be in Basic, but here I use Pascal and Fortran because they are better adapted to Scientific Programming. The different languages in vogue each have their strengths and weaknesses. These must be judged in the light of the application to which they are to be put. Once you have learnt one language, you will find it easier to learn a second.]

In Pascal we might have

```
program inverse(input, output);
var
        x, y:real;
begin
readln(x);

if x <> 0.0 then
        begin
        y:=1.0/x;
        writeln(x,y)
        end
else
        writeln('Error:Input=0')
end.
```

Alternatively, in Fortran we could write:

```
program invers
implicit none
real x,y
read(5,*) x

if (x.ne.0.0) then
        y=1.0/x
        write(6,*) x,y
else
        write (6,*) 'Error:  Input=0'
endif
stop
end
```

The first summation algorithm can easily be modified to give a factorial algorithm:

```
READ n
SET factorial TO 1
FOR i = 1 TO n DO
        REPLACE factorial BY factorial × i
WRITE n, factorial
```

Converting this to Fortran:

```
program factorl
implicit none
integer n, nfac, i
read*, n
nfac = 1
do i=1,n
        nfac=nfac*i
enddo
write (6,*) n,nfac
stop
end
```

There are some points to explore about this program, all to do with the fact that real numbers (rather than integers) give only an approximation, but they do postpone overflow.

i) This works fine for small n. What happens when $n = 20$?
ii) Try making *nfac* a real number. Does this work for all values of n?
iii) Try to get an overflow.

Calculating an approximation to the limit of convergent series of the form

$$S = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots\dots + \frac{1}{n^2} + \dots$$

is possible if we use an "indefinite loop", such as the following algorithm illustrates:

```
READ tolerance
SET i TO 0
SET sum TO 0
REPEAT
        REPLACE i BY i+1
        REPLACE sum BY sum+1/i^2
UNTIL |1/i^2| < tolerance
WRITE i, sum
```

(The result from this algorithm approximates $\pi^2/6$, the value known to be approached as more and more terms are taken.) Here we see that the indented operations are repeated until the size of $1/i^2$ is less than some specified tolerance. Note that there is not always a clear relation between the tolerance and the actual error! The tolerance tells us how quickly the succeeding terms are changing, **not** how close the sum is to a converged value. It tells us in this instance about $1/i^2$ rather than about

$$\left|\frac{\pi^2}{6} - \left(\frac{1}{1^2}+\frac{1}{2^2}+\dots+\frac{1}{n^2}\right)\right|$$

Also, just because the terms are getting smaller doesn't always mean that the series will converge. For example, consider the harmonic series

$$\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\dots\dots+\frac{1}{n}$$

The terms indeed decrease, and approach zero, but the sum is unbounded and approaches ∞ as n approaches ∞.[1]

For more complicated series, such as $1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\dots\dots+\frac{x^n}{n!}+\dots$,

for a specified value of x, the loop can also be used to evaluate each term:

```
READ x
SET i TO 0
SET term TO 1
SET sum TO 1
REPEAT
```

[1] A proof of this was given in *Function*, *Vol* 1, *Part* 4 back in 1977.

```
        REPLACE i BY i + 1
        REPLACE term BY term × x/i
        REPLACE sum BY sum + term
UNTIL |term| < tolerance
WRITE tolerance, i, sum
```

In this case we use the value of *sum* and *term* from the last time through the loop, and simply increment them. This is much more efficient than calculating x^i separately for each value of i.

The series involved here converges to the value of e^x. [e is a number important to the whole of the number system. It is usually described as the base of the natural logarithms.] We will write a program to evaluate e by setting $x = 1$ in this series. This gives

$$e = 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \ldots.. + \frac{1}{n!} + \ldots\ldots .$$

We will continue adding terms until the next term to be added has a magnitude less than 10^{-6}. In Pascal, we might write:

```
program findsum(output);
const
        tolerance = 1.0e-6;
        imax=100;
var
sum,term:real;
i:integer;
begin
i:=0;
term:=1.0;
sum:=term;
repeat
        i:=i+1;
        term:=term/i;
        sum:=sum+term;
until (abs(term) < tolerance) or (i=imax);
writeln(sum)
end.
```

Alternatively , in Fortran we might have:

```
program findsum
implicit none
integer imax,i
parameter (imax=100)
real tol,sum,term
parameter (tol=1.e-6)
i=0
term=1.0
sum=term
do while (abs(term).ge.tol.and.i.lt.imax)
        i=i+1
        term=term/i
        sum=sum+term
enddo
write(6,*) sum
stop
end
```

With these codes as templates, you should be able to do these exercises in either Pascal or Fortran:

1. Read an integer n, and then use the "if-then-else" structure to determine if the integer is odd or even. Write the word "odd" or "even" to the screen, as appropriate.

2. Read two values tol and x. Use a "repeat until" structure to calculate e^x from the series given, by including all the terms which are larger than tol. Write the result to the screen.

3. The sum $f(n)=\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+.......+\frac{1}{n^2}$ converges to $\frac{\pi^2}{6}$ as n tends to infinity. Write a program to compute $f(n)$ for n = 10,000 by computing the sum from n = 1 to 10,000. Then repeat, but do the addition in reverse order (from 10,000 to 1). What do you notice? Explain the result. (Hint: See *Function*, *Vol* **16**, *Part* **5**, pp 147-149.)

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OLYMPIAD NEWS

Hans Lausch, Monash University

The 2002 Australian Mathematical Olympiad

The Australian Mathematical Olympiad (AMO) was held in Australian schools on February 12 and 13. On either day, 98 students in years 8 to 12 sat a paper consisting of four problems. For which they were given four hours. These are the two papers. No calculators were allowed, and each question was worth seven points.

Paper 1

1. Let m and n be positive integers such that $2001m^2 + m = 2002n^2 + n$ holds. Prove that $m - n$ is a perfect square.

2. Determine all triples (u, v, w) of real numbers satisfying:
   (i) $u + v + w = 38$
   (ii) $uvw = 2002$
   (iii) $0 < u \leq 11$, $w \geq 14$.

3. A line through a vertex of a triangle is called a dividend if it cuts the triangle into two triangles of equal perimeters.

   Let ABC be a triangle. Prove that the dividends of ABC are concurrent.

4. Determine the largest positive integer n for which there exists a set S with exactly n numbers such that

   (i) each number in S is a positive integer not exceeding 2002;
   (ii) if a and b are two (not necessarily different) numbers in S, then their product ab does not belong to S.

Paper 2

5. Determine all real-valued functions f defined for all real numbers which satisfy

$$f(2002x - f(0)) = 2002x^2$$

for each real x.

6. Let $ABCD$ be a rectangle, let E be a point on BC and F a point on CD such that AEF is an equilateral triangle.

Prove that the area of triangle ECF equals the sum of the areas of triangle ABE and triangle AFD.

7. Let n and q be integers, $n \geq 5$, $2 \leq q \leq n$. Prove that $q-1$ divides $\left\lfloor \frac{(n-1)!}{q} \right\rfloor$.

(Note: $\lfloor x \rfloor$ is the largest integer not exceeding x.)

8. Let a, b, c be real numbers, and suppose that A, B, C, D are real numbers such that

$$(Ax + B)(Cx + D) = ax^2 + bx + c$$

for all real numbers x.

Prove that at least one of the numbers a, b, c is greater than or equal to $\frac{4}{9}(A + B)(C + D)$.

As a result, 36 students were invited to represent Australia at the Fourteenth Asian Pacific Mathematics Olympiad (APMO). This annual competition was started in 1989 by Australia, Canada, Hong Kong and Singapore. Since then the APMO has grown into a major competition for students from about twenty countries on the Pacific rim as well as from Argentina, South Africa and Trinidad & Tobago. It was held on March 11. Here is the contest paper.

[Time allowed: 4 hours; No calculators may be used; Each question is worth 7 points.]

Problem 1.

Let $a_1, a_2, a_3, \dots, a_n$ be a sequence of non-negative integers, where n is a positive integer. Let

$$A_n = \frac{a_1 + a_2 + \dots + a_n}{n}$$

Prove that

$$a_1!a_2!\dots a_n! \geq (\lfloor A_n \rfloor!)^n,$$

where $\lfloor A_n \rfloor$ is the greatest integer less than or equal to A_n, and

$$a! = 1 \times 2 \times \dots \times a \text{ for } a \geq 1 \text{ (and } 0! = 1).$$

When does equality hold?

Problem 2.

Find all positive integers a and b such that

$$\frac{a^2+b}{b^2-a} \quad \text{and} \quad \frac{b^2+a}{a^2-b}$$

are both integers.

Problem 3.

Let ABC be an equilateral triangle. Let P be a point on the side AC and Q be a point on the side AB so that both triangles ABP and ACQ are acute. Let R be the orthocentre of the triangle ABP and S the orthocentre of triangle ACQ. Let T be the point common to the segments BP and CQ. Find all possible values of $\angle CBP$ and $\angle BCQ$ such that triangle TRS is equilateral.

Problem 4.

Let x, y, z be positive numbers such that

$$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1.$$

Show that $\sqrt{x+yz}+\sqrt{y+zx}+\sqrt{z+xy}\geq\sqrt{xyz}+\sqrt{x}+\sqrt{y}+\sqrt{z}$.

Problem 5.

Let **R** denote the set of all real numbers. Find all functions f from **R** to **R** satisfying:

(i) there are only finitely many s in **R** such that $f(s)=0$, and
(ii) $f(x^4+y)=x^3f(x)+f(f(y))$ for all x, y in **R**.

Following the APMO, 25 students were invited to take part in a training school held in Sydney from April 13 to 23. About half the students had qualified themselves as candidates for the team that is to represent Australia in this year's International Mathematical Olympiad (IMO), while the remaining students were younger and had distinguished themselves by their excellent performance in both AMO and APMO. Team candidates had to sit two more tests (IMO selection tests), and subsequently this year's Australian IMO team that will compete in Glasgow during July with teams from some 80 countries was selected.

This is the 2002 Australian IMO team; all team members are Year 12 students.

David Chan, Sydney Grammar School, NSW
Andrew Kwok, University High School, Vic
Nicholas Sheridan, Scotch College, Vic
Gareth White, Hurlstone Agricultural High School, NSW
Stewart Wilcox, North Sydney Boys' High School, NSW
Yiying Zhao, Penleigh and Essendon Grammar School, Vic.

The reserve is Elena Kelareva, Elizabeth College, Tas.

Congratulations to all, and our best wishes for Glasgow!

PROBLEMS AND SOLUTIONS

We begin by giving the solutions to the problems posed in last October's issue.

SOLUTION TO PROBLEM 25.5.1 (submitted by Peter Grossman)

The problem read:

Given any four-digit number in which not all the digits are the same, a sequence is generated as follows. The first term is the given number, and each term is used to determine the next term according to the following rule:

1. Rearrange the digits of the number into decreasing order to obtain a four-digit number *ABCD*, and into increasing order, to obtain another four-digit number *DCBA* (where *A*, *B*, *C*, *D* denote the digits).
2. Subtract *DCBA* from *ABCD*.

(For example, starting with 5946, the sequence 5946, 5085, 7992, 7173, 6354, 3087, 8352, 6174, 6174, ... is generated.)

Prove that the sequence must eventually reach 6174, regardless of the given number.

Solutions were received from Keith Anker, Garnet J Greenbury, Joseph Kupka, Carlos Victor and the proposer.

Kupka notes the possibility of a "barbarous" solution in which all 9990 possible cases were checked individually by a computer program. Greenbury notes that the second term must be a multiple of 9 (we leave readers to prove this fact for themselves) and that therefore only 1110 cases remain to be checked. If we further consider that permutations of the digits are irrelevant, then we may reduce the size of the task even further (to below 50 cases). However, initial zeroes must always be included.

But we may do better than this by analysing the problem in rather more detail. This was the course chosen by most of our solvers, although Kupka referred to the result as "quasi-barbarous"! All the solutions reduce to the discussion of a list of different cases. Here we follow the solution submitted by the proposer.

"Consider the calculation of the second term from the first. By construction, $A \geq B \geq C \geq D$, and $A > D$. Let the result $ABCD - DCBA$ be the four-digit number $WXYZ$. There are two cases to consider.

Case 1: $B > C$. By analysing the subtraction process, we obtain the following results:

$$Z = D - A + 10 \qquad (1)$$
$$Y = C - B + 9 \qquad (2)$$
$$X = B - C - 1 \qquad (3)$$
$$W = A - D \qquad (4)$$

Equations (1) and (4) yield $W + Z = 10$. Equations (2) and (3) yield $X + Y = 8$.

Case 2: $B = C$. In this case, we obtain:

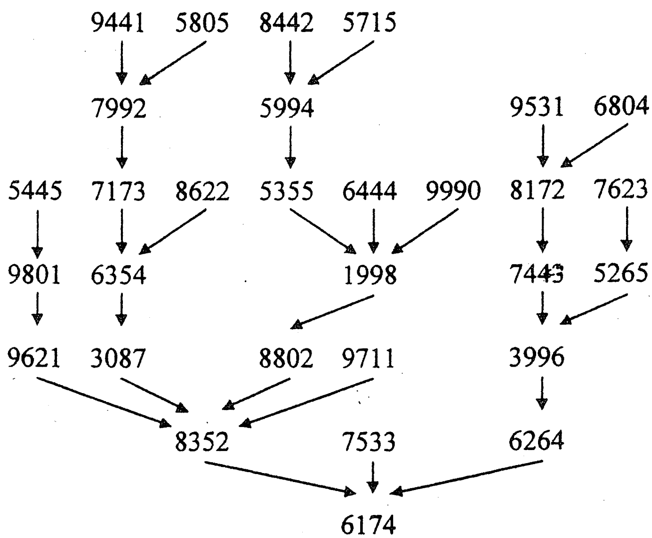
$$Z = D - A + 10 \qquad (5)$$
$$Y = 9 \qquad (6)$$
$$X = 9 \qquad (7)$$
$$W = A - D + 1 \qquad (8)$$

Equations (5) and (8) yield $W + Z = 9$.

"Case 1 yields five possible pairs of values of W and Z (disregarding order): (1, 9), (2, 8), (3, 7), (4, 6), (5, 5), and five possible pairs of values of X and Y: (0, 8), (1, 7), (2, 6), (3, 5), (4, 4). Therefore there are 25 possible values of $WXYZ$, disregarding permutations of the digits.

"Case 2 yields five pairs of values for W and Z: (0, 9), (1, 8), (2, 7), (3, 6), (4, 5). Since the values of X and Y are fixed, five values of $WXYZ$ arise from this case, again disregarding permutations of the digits.

"Therefore there are 30 possibilities for the second term of the sequence. We now need to show that all of these 30 numbers ultimately yield 6174 when successive terms of the sequence are generated from them. The result follows the diagram below, in which all 30 numbers appear. An arrow from one number to another indicates that the second number is generated from the first by applying the rule. Starting at any number in the diagram and following the arrows, we eventually reach 6174. Hence 6174 or a permutation of its digits must be reached starting from any four-digit number. To finish the proof, it only remains to observe that, if the rule is applied to any permutation of the digits of 6174, the number 6174 itself is obtained."



[This is a famous problem, usually attributed to the amateur Indian mathematician D R Kaprekar. Indeed, the number 6174 is sometimes referred to as "Kaprekar's Number". Kaprekar twice wrote for *Function* (in *Volume* **8**, *Part* **2** and in *Volume* **9**, *Part* **1**). Readers may care to explore what happens with 2-digit and 3-digit numbers; very dedicated readers may care to examine the case of 5-digit numbers!

It follows from the details of Grossman's proof that it takes at most seven steps to reach the endpoint of 6174; for the numbers listed are the possibilities for the second member of the sequence, and the longest chains in the diagram above have six links.]

SOLUTION TO PROBLEM 25.5.2 (submitted by Peter Grossman)

The problem read:

Find all real number solutions in x and y of the equation $x^y = 1$.

Solutions were received from Keith Anker, J A Deakin, Carlos Victor and the proposer. With two important provisos, discussed below, all were the same, and we here print a composite.

Consider first the case in which $x > 0$. Then, taking the natural logarithm of both sides, we find $\ln(x^y) = 0$. Thus $y \ln x = 0$, and it follows that either $y = 0$ or $x = 1$.

Now consider $x < 0$. The proposer's solution took it that the expression x^y required y to be an even integer if it were to be real and positive. In this case, we have $x = -1$, $y =$ an even integer, or else $y = 0$. Deakin also took this view. Anker allowed the case in which y was a fraction with an even numerator and an odd denominator. In this case, $(-1)^y$ will be multivalued, but one of its values is 1.

The final possibility is $x = 0$. Anker allowed the case $x = y = 0$. The other solvers did not. [For a discussion of the value to be assigned to the expression 0^0, see *Function*, August 1981. Many authorities assign a value of 1 to this expression, while others treat it as undefined.]

Summarising therefore, we have the following possibilities for (x, y): (1, anything), (anything except 0, 0), (–1, any even integer), and possibly (0, 0) and (–1, any fraction with even numerator and odd denominator).

SOLUTION TO PROBLEM 25.5.2 (submitted by Julius Guest)

The problem asked for the sum of the series

$$\frac{2}{1!} + \frac{9}{2!} + \frac{28}{3!} + \ldots\ldots\ldots + \frac{n^3+1}{n!} + \ldots\ldots$$

Our printed version was marred by a misprint in that the general term was displayed with a minus sign instead of a plus. Nonetheless, we had solutions from J A Deakin, Carlos Victor and the proposer. (Deakin indeed solved the problem both ways.) Here is Victor's elegant solution.

Because $\dfrac{n^3+1}{n!}=\dfrac{n^2}{(n-1)!}+\dfrac{1}{n!}$ we may write the sum as

$$2+\frac{9}{2}+\left(\frac{3^2}{2!}+\frac{4^2}{3!}+\ldots\ldots+\frac{n^2}{(n-1)!}+\ldots\ldots\right)+\left(\frac{1}{3!}+\frac{1}{4!}+\frac{1}{5!}+\ldots\ldots\right)$$

But now $\dfrac{n^2}{(n-1)!}=\dfrac{(n-1)(n-2)+3(n-1)+1}{(n-1)!}=\dfrac{1}{(n-3)!}+\dfrac{3}{(n-2)!}+\dfrac{1}{(n-1)!}$ and so

the sum to be evaluated becomes

$$2+\frac{9}{2}+\left(1+\frac{1}{1!}+\frac{1}{2!}+\ldots\ldots\ldots\right)+3\left(\frac{1}{1!}+\frac{1}{2!}+\ldots\ldots\right)+\left(\frac{1}{2!}+\frac{1}{3!}+\ldots.\right)+\left(\frac{1}{3!}+\frac{1}{4!}+\frac{1}{5!}+\ldots\ldots\right)$$

But it is known that $1+\dfrac{1}{1!}+\dfrac{1}{2!}+\dfrac{1}{3!}+\ldots\ldots\ldots=e$, and so the series sums to

$2+\dfrac{9}{2}+e+3(e-1)+(e-2)+\left(e-\dfrac{5}{2}\right)$ which simplifies to $6e-1$.

SOLUTION TO PROBLEM 25.5.2

This problem came from the *International Journal of Mathematical Education in Science and Technology* and asked for a proof that 7 divides the number $abcde$ (in decimal notation) if and only if it also divides the number $abcd-2e$.

Solutions were received from Garnet J Greenbury, Paul Grossman, David Halprin, J A Deakin and Carlos Victor. Here is Grossman's solution.

"The number $abcd$ as defined is non-negative and no larger than 9999. There is no reason why we should not replace it with N, which may be any integer. To avoid confusion, let us give the digit e in the problem the notation u (for units), an integer in the range 0 to 9.

"The problem then reads: Prove that 7 divides the number $10N + u$ if and only if it divides the number $N - 2u$ (Statement 1).

"Further to the expressions $A = 10N + u$, $B = N - 2u$, let us introduce another expression $C = 3N + u$. We shall show that:

7 divides A if and only if it divides C (Statement 2)
7 divides B if and only if it divides C (Statement 3)

If these two statements are true then Statement 1 must be true.

"Proof of Statement 2: $A = 10N + u = 7N + 3N + u = 7N + C$. Thus A and C differ only by a multiple of 7 and the statement must hold.

"Proof of Statement 3: $3B = 3N - 6u = C - 7u$. Multiplying by 3 and adding a multiple of 7 has no effect on divisibility by 7 and the statement must hold."

This problem led Dr Grossman to a reminiscence.

"The result brings back memories of a distant time and a distant land. In the 1930s in Central Europe pupils in a primary school were taught how to tell whether 2 or 3, 4, 5, 6, 8, 9, 10 or 11 divide a given number. One boy wondered why there was no test for the number 7. He found with two-digit numbers that multiplying the first digit by 3 and adding the second gave him a smaller number which, if divisible by 7, would show that the original number was divisible by 7. It worked with numbers of more than two digits too, but became rather tedious. He did not have the experience to write a proof as Statement 2 above, but with some mental effort convinced himself that the rule had to hold. His teacher did not think much of the idea, arguing that it was quicker to perform the division by 7.

"The boy could have expanded the process by deriving the sequence 1, 3, 2, 6, 4, 5 recurring to allow the units of the given number to be multiplied by 1, the tens by 3, the hundreds by 2 and so on. Readers might like to verify the validity of this procedure. But I think the teacher had a point when he thought division was faster. None of this suggests that there is something special about the number 7. It is purely in our decimal system that we have no easy way to tell if 7 divides into a given number."

And now for some new problems.

PROBLEM 26.1.1 (the third, last and hardest of the "Professor Cherry problems", this time from p 116 of Todhunter's *Algebra*; see the note to Problem 26.1.1)

Show that

$$\frac{d^m(a-b)(b-c)+b^m(a-d)(c-d)}{c^m(a-b)(a-d)+a^m(b-c)(c-d)}=\frac{b-d}{a-c}$$

when $m = 1$ or 2.

PROBLEM 26.1.2 (a classic, posed in our Computer column last February)

Let a circular field of unit radius be fenced in, and tie a goat in its interior to a point on the fence with a chain of length r. What length of chain must be used in order to allow the goat to graze exactly one half the area of the field?

PROBLEM 26.1.3 (originally posed in the "Ask Marilyn" column of the US magazine *Parade*; much reproduced since)

Mr and Mrs Smith are in the habit of dining two nights each (seven-day) week at the Taste-e-Bite Café. It is quite random which two nights they choose, but about three quarters of the time they notice that Mr and Mrs Brown are also there. They conclude that the Browns eat there more frequently than they do themselves. Is their conclusion justified?

PROBLEM 26.1.4 (from *Mathematical Fallacies, Flaws and Flimflam*, Ed E J Barbeau)

Without recourse to tables, calculators or computers, find the value of $\log_3 169.\log_{13} 243$.

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Published by School of Mathematical Sciences, Monash University