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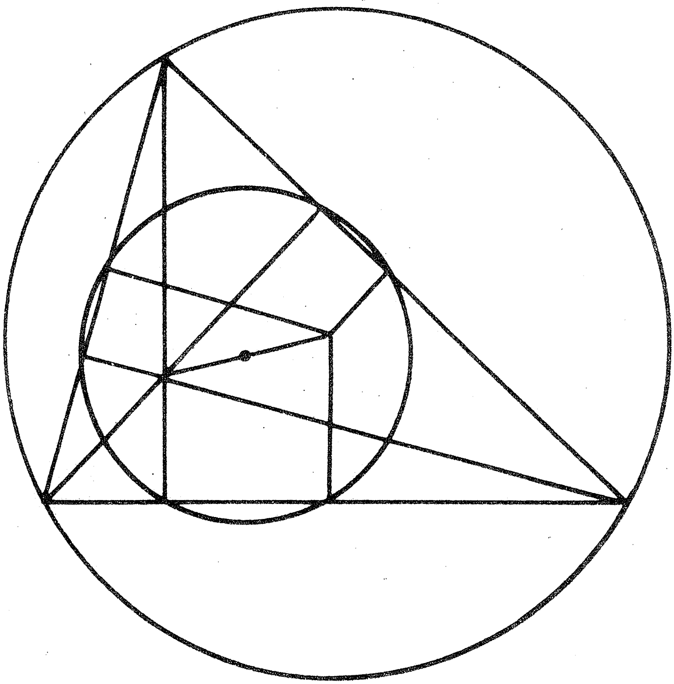

A SCHOOL MATHEMATICS MAGAZINE

Published by Monash University

Function is a mathematics magazine addressed principally to students in the upper forms of schools. Today mathematics is used in most of the sciences, physical, biological and social, in business management, in engineering. There are few human endeavours, from weather prediction to siting of traffic lights, that do not involve mathematics. *Function* contains articles describing some of these uses of mathematics. It also has articles, for entertainment and instruction, about mathematics and its history. Each issue contains problems and solutions are invited.

It is hoped that the student readers of *Function* will contribute material for publication. Articles, ideas, cartoons, comments, criticisms, advice are earnestly sought. Please send to the editors your views about what can be done to make *Function* more interesting for you.

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Articles, correspondence, problems (with or without solutions) and other material for publication are invited. Address them to:

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Alternatively correspondence may be addressed individually to any of the editors at the addresses shown above.

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With this issue, *Function* enters its third year of publication. As it is a journal for senior school students, we aim to make our material interesting for this set of readers. Three long articles of general interest appear here: Charles Johnson's account of the pioneering history of the computer, a biography of one of Australia's greatest women mathematicians, and a further instalment in Peter Finch's historical studies in statistics.

The solution of difficult and elegant problems is an important part of all branches of mathematics. This issue contains a further set of problems which we hope will interest and involve our readers. We also print solutions to some of the earlier problems, but leave some of the more interesting ones open in the hope that readers will send us their answers.

The editors welcome correspondence from readers, particularly school student readers. Please send us solutions to problems, letters, articles, book reviews or general comments. Feedback from our readers helps us to make *Function* a more attractive and readable magazine. Please let us know what you think of it, and send us your suggestions for articles you would like to read, topics you would like covered and problems you would like solved.

It is our policy to give precedence to articles by school students and many such articles have already appeared. It is up to you, our readers, to help us maintain this policy.

CONTENTS

THE FRONT COVER

M.A.B. Deakin, Monash University

Many elegant and unexpected theorems may be proved to hold for the triangle. A number of these are shown on the diagram opposite. If ABC is any triangle, we may draw perpendiculars from each vertex to its opposite side. These perpendiculars meet the sides in the points D,E,F. Our first theorem is that the perpendiculars all intersect at a common point H.

If A',B',C' are the mid-points of the sides, and perpendiculars are constructed at these points, these lines all meet at a common point O. Now let N be the point half-way between O,H.

With centre N and radius ND, draw a circle. This circle passes through D,E,F and also through A',B',C'. It also passes through points N_a,N_b,N_c, the mid-points of AH,BH,CH respectively. It is thus referred to as the *nine-point circle*.

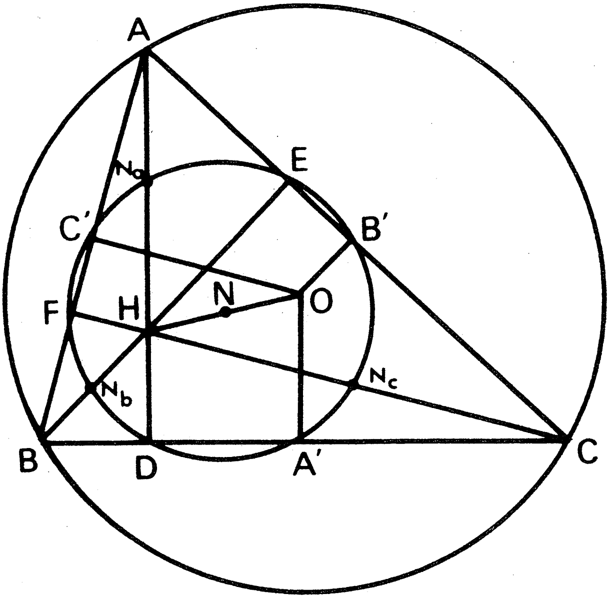
With centre O and radius OA, draw a circle. This passes through the points A,B,C and is referred to as the *circumcircle*. The radius of the circumcircle is exactly twice that of the nine-point circle.

Exactly four circles (the *equicircles*) may be drawn such that AB,BC,CA are tangents to all four. One (the *inscribed circle*) lies within the triangle; the others (*escribed circles*) lie outside it. Each of these circles is tangent to the nine-point circle.

You may care to try to prove some of these results. They hold for all triangles, even though, if ABC has an obtuse angle, the point H lies outside the triangle. Some special cases lead to interesting situations. What happens if ABC is an equilateral or right-angled triangle?

The history of these theorems is an interesting story, too. The fact that AD, BE and CF all pass through the common point is one of the more difficult of the classical (ancient Greek) results. The discovery that the perpendiculars at A',B',C' all pass through the common point O is also classical, and rather easier to prove.

The question of who gets the credit for the discovery of the nine-point circle is a more difficult one. It is sometimes attributed to L. Euler (1707 - 1783), one of mathematics' superstars. In this case, however, the evidence is against him. There seem to have been several independent discoveries, but all rather later. The earliest documented account of the theorem is due to an otherwise obscure Englishman, Benjamin Bevan, who published the result in 1804, but the proof was not forthcoming until a later paper by Butterworth (1806).



Many modern authors attribute the result to two French geometers, Brianchon and Poncelet, whose discovery is almost certainly independent, but quite certainly later (1821). The tangency of the nine-point circle to the four equicircles was first established by the German geometer Feuerbach in 1822, but no strictly geometric proof appeared until 1850.

It may well be that further theorems await discovery. Geometry was taught till some 30 years ago in our schools, and some syllabuses included the above theorems. Recently, several eminent mathematicians, pre-eminently the French geometer, René Thom, have suggested that geometry should be reinstated into school syllabuses, at the expense of some of the more abstract, symbolic components.

What do our readers think?

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Only those topics which have a quality of "play" have educational value, and of all such games, Euclidean geometry, with its constant references to underlying intuitively understood fundamentals, is the least gratuitous and the richest in meaning.

René Thom.

BABBAGE AND THE ORIGINS OF COMPUTERS

C. H. J. Johnson, C.S.I.R.O. Division of Chemical Physics

The programmable pocket calculator which is now so readily available is an immediate descendant of the modern large scale computer with its very much greater computing power. This in turn has its origins in the mechanical calculators of the last century and indeed in those of earlier centuries. The idea of using a train of gear wheels, linked so that each time one wheel completes a revolution, the next wheel turns so as to record the "carry" of one unit, is very old and appears in the writings of Heron of Alexandria in the first century A.D. However, it was not until the early seventeenth century that the idea of using such a gear train to construct an adding machine first appeared. Pascal (1642) developed an adding machine for accounting work but the first successful calculator that could add, subtract, multiply and divide, seems to have been built by Leibnitz in 1695. These early machines were not mechanically reliable, but as technology improved more and better machines appeared, although commercially successful calculators did not appear until about the middle of the nineteenth century when the business world created a demand for such machines.

Until effective desk calculators became available, all large calculations were done by hand using logarithms for multiplications. In particular, calculations in astronomy were done this way, a prime example being J.C. Adams's mathematical discovery of the planet Neptune in 1845. It seems obvious to us now that machines could be made to do this labour but the first man to investigate this possibility seriously was Charles Babbage who in 1821 designed what he called a "difference engine" which consisted of a set of linked adding mechanisms, capable of generating successive values of an algebraic function by the method of finite differences†. After considerable effort he built a small machine that used only first and second order differences† and which clearly demonstrated

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†Consider the values $y = f(x)$ of the function f defined on some interval [0,1], say. Divide the domain [0,1] up into n equal intervals by the points $x_0 = 0$, $x_1 = \frac{1}{n}$, ..., $x_n = 1$; so that $x_i - x_{i-1} = \frac{1}{n}$, for $i = 1,2,\dots,n$. Write $y_i = f(x_i)$, for $i = 1,2,\dots,n$. Then we define the *first differences* of f, corresponding to this subdivision of [0,1] as the differences $y_i - y_{i-1}$, for $i = 1,2,\dots,n$. We write Δy_i for $y_i - y_{i-1}$. The *second differences* of f are denoted by $\Delta^2 y_i$ and are defined by $\Delta^2 y_i = \Delta y_i - \Delta y_{i-1}$; and so on for third differences, etc. The *method of finite differences* referred to here is a method of approximating the value of a function from a knowledge of its first, second, etc., differences. If the function is a polynomial

the feasibility of his ideas. Babbage invented his machines at a time when precision mechanical engineering was not of a sufficiently high standard to make the elaborate machinery he needed, so that he was faced at all times with the double problem not only of designing his machinery but also of constructing the tools for making it. It is perhaps not surprising, then, that attempts to make a full-scale difference engine did not succeed at the time. Babbage also designed a program-controlled mechanical machine that would perform an extended sequence of arbitrary arithmetic operations and print out the results. This was his "Analytical Engine". This machine was never built although parts of it were made eventually. A very great deal of Babbage's scheme for the analytical engine is now implemented in terms of modern technology and Charles Babbage is really the originator of the computer as we know it today. Babbage's failure to achieve any real success was due both to the inadequacies of the technology of the day and to the general lack of appreciation of what he was trying to do - he was just too far ahead of his time.

Babbage's earliest recollections of his original idea for making an automatic computer are contained in his autobiography "Passages from the Life of a Philosopher" where he first mentions the possibility of computing tables of logarithms by machinery. This was about 1812 but he did not begin serious work until around 1820 when his ideas took on a more definite form. He observes:

"I considered that a machine to execute the more isolated operations of arithmetic would be of comparatively little value, unless it were very easily set to do its work, and unless it executed not only accurately, but with great rapidity, whatever it was required to do."

He then goes on to describe the way in which tables might be constructed using the method of finite differences. Thus,

"On the other hand, the method of differences supplied a general principle by which all Tables might be computed through limited intervals, by one uniform process. Again, the method of differences required the use of mechanism for Addition only."

He adds that the machine must be able to set up the tables in type ready for printing.

In this way Charles Babbage conceived the idea of a mechanical calculator and then designed his difference engine which would perform his calculations automatically. This was all very new and the mathematical method - finite differences - was one that had never been used before in a mechanical calculator. Of course, the method only applies to functions which behave locally as polynomials, and Babbage was well aware of this. With his model difference engine Babbage was able to compute tables of squares, triangular

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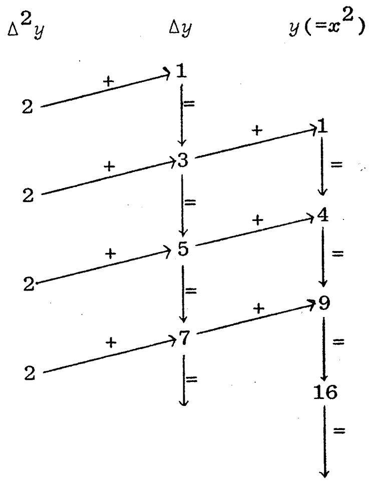
then it can be calculated directly from its differences. Indeed a function is a polynomial of degree k if and only if its $(k + 1)$-th differences are zero whatever subdivision into equal intervals of the domain is made. The method of finite differences effectively approximates a function by a polynomial of appropriate degree. [For further information the reader can consult Volume II of Durell and Robson's *Advanced Algebra*, Chapter X.]

numbers, and successive values of the polynomial $x^2 + x + 41$, a quadratic function whose first thirty-nine values are all prime numbers. (See *Function* Volume 1, Problem 5.4 and its solution in Volume 2, Part 3.) In his work with computing machines Babbage was always concerned that the time for basic operations should be successively reduced. Indeed, he constantly pointed out that there is little point in constructing a machine unless it performs calculations very much more rapidly than a human calculator. In a letter to Humphrey Davey, Babbage comments on the calculation of tables of logarithms and sines to be computed by order of the French government, computations to twenty significant figures over a range that made a total of eight million digits, and says that with a difference engine working up to sixth differences, less than a dozen operators would be needed, as opposed to the French requirement of ninety-six, and furthermore, the dozen would get the correct answers.

Let us now consider the mathematical principle of the difference engine. The machine Babbage actually constructed worked with second differences and so solves the equation $\Delta^2 y_n = c$, where c is a constant. The general solution is $\frac{1}{2}cn^2 + an + b$, where a and b are arbitrary constants. If now y_0 and y_1 be given, so that Δy_0 can be calculated, and hence a and b can be given unique values, the whole sequence of values of y follows immediately, as shown in the figure where a table of squares is constructed. Note that the figure illustrates the case for which $a = b = 0$ and $c = 2$.



The great advantage of the method of finite differences is that only addition is required, and further, not only can the tabular values be computed very rapidly, but the solution can be checked at any stage, with the virtual guarantee that if the current result is correct, so also are all the previous ones. As far as the construction of tables was concerned, all functions like log, tan and sin, can be approximated locally by polynomials of sufficiently high degree, so that if a polynomial of the mth degree is required, then we must use up the mth differences.

Babbage's first difference engine was such a success that the Royal Astronomical Society gave him a gold medal and the Chancellor of the Exchequer agreed that the difference engine in its general form was worthy of Government support. In 1823 work began on the construction of the difference engine and continued steadily for four years. However, by 1827 expenses were far greater than anticipated and work virtually ceased. Governments changed and the project foundered, because, as the Earl of Rosse said in his presidential address to the Royal Society in 1854, "there was no tangible evidence of immediate profit". A final irony - in 1834 a Stockholm printer named Sheutz heard of Babbage's engine and produced a simpler version. In 1864 the British Government bought a copy of the Sheutz machine for computing life contingency tables.

Almost as soon as he started to build his difference engine, Babbage became dissatisfied with its limitations, perceiving that "mechanism" could be used to solve very much more general problems. He developed mechanisms for multiplication and division, and saw that mechanism could provide a means not only for performing a sequence of elementary operations but also for controlling this sequence. He was thus led to the Analytical Engine, a computing machine whose computational processes were to be implemented entirely in mechanical terms. In his concept of a computing machine Babbage not only far outpaced the ideas of his contemporaries but also the technology of his time and so the history of the analytical engine is rather briefer than that of the difference engine. Despite all his efforts - and encouragement from other eminent mathematicians of the day - the analytical engine was never anywhere near completed, although several fragments were completed long after his death. As with the difference engine, Babbage published no systematic account of its principle and working. His paper "On the Mathematical Powers of the Calculating Engine", published at the end of 1837 describes the machine in very general terms. Some account appears in his autobiography and more detailed accounts are given in L.F. Menebraea's "Sketch of the Analytical Engine" and translated into English, with copious notes added, by Ada Augusta King, Lady Lovelace, Lord Byron's daughter. These notes provide the most detailed account of the mathematical workings of the Analytical Engine available. The account of how Babbage came to the idea of the analytical engine is given by his son H.P. Babbage. "The idea of the analytical engine arose thus - When the fragment of the difference engine now in the museum was put together early in 1833, it was found that, as had been before anticipated, it possessed powers beyond those for which it was intended, and some of these could be, and in fact were, demonstrated with this fragment. It is evident that by interposing a few connecting wheels, the column of the result can be made to influence the last difference, or any other part of the machine in several ways. Following out this train of thought, he first proposed to arrange the axes (i.e. shafts) of the Difference Engine circularly, so that the Result column should be near the last difference, and thus easily within reach of it. He called this arrangement 'the machine eating its own tail' (and is in fact a form of differential analyser). But this soon led to the idea of controlling the machine by entirely independent means, and making it perform not only addition, but all the processes of arithmetic at will in any order and as many times as required."

The design of the analytical engine was that it should consist of a store in which numbers could be set up and held in counters (registers), some thousand in all, each capable of holding a fifty digit number, a mill or central processor, in which arithmetic operations were to be carried out on numbers taken from the store and a control unit for determining the sequence of operations. The control of the machine was to be carried out by means of punched cards, rather like those that were then used for the control of Jacquard looms in making elaborate laces. Plungers passing through holes in the cards were to operate mechanisms for bringing numbers from the store, initiating arithmetic operations and so on. There were to be several types of control cards. First, the variable cards which specified which numbers (variables) were to be taken into the mill and the register to which the result was to be transferred. Second, the operation cards which specified the particular operation to be performed. Each variable which resided in the store was given a name, a letter, and these appeared on the variable cards. It follows, that each "instruction" requires three "addresses" for its operation, so in modern terms, Babbage's machine may be considered a three-address machine. For a given sequence of arithmetic operations, the corresponding set of instruction cards constituted a formula and at any later time the operation of a given set of cards could be repeated, so recalculating the formula, with possibly different values of the variables. That is, the analytical engine was to possess a library.

There arose the question of what the machine would do, if in the midst of algebraic operations, it was required to perform logarithmic or trigonometric operations. These functions were to be provided by tables set on punched cards - the table cards - these having been computed by the engine and "would therefore be correct". A computing facility in the store was to provide interpolated values. If the machine wanted a tabular number, a logarithm, for example, then it would ring a bell and stop itself. On this, an attendant would look at an indicator and find that the log of 2303 was required. He would then place the appropriate card in the machine, which would inspect the argument punched on the card to see that it was the required one. If not it would ring a louder bell than before and stop, otherwise it would proceed. Babbage was very concerned that the number of variables, operations and numerical constants used by the engine should be without limit and said that he had "replaced infinity in space by infinity in time". He was very conscious of working with finite means.

One of the important elements of Babbage's design of a computer was that the machine should be capable of "judgement", so that in the course of a calculation when two or more courses presented themselves, the machine should be able to select the appropriate one, especially when the proper course to be adopted could not be known in advance. Babbage illustrated this "judgement" facility by discussing an algorithm for finding the real zeros of a polynomial of arbitrarily high degree. First, the number of real zeros is known from an application of Sturm's theorem* and from this, starting at a number greater than the

*See *Advanced Algebra*, Volume II, Chapter XIII, referred to in the previous footnote.

largest zero, a process of successive subtabulation will serve to locate each zero. The passage through a zero is readily ascertained by observing the change in sign in the value of the polynomial and this was to be done by detecting the carry in the highest digit in the register receiving the value, that is, by observing overflow. This effect can be made to work a lever which can cause a change in the course of events.

Perhaps, it is not surprising that Babbage's Analytical Engine was never built - he was much too early. Babbage's machines were entirely mechanical - electrical relays were invented by Henry around 1835 but were not commercially available until very much later. However, Babbage was at all times aware of the limitations of his machines and, in particular, of making the speed as great as possible. For the Analytical Engine, he reckoned on a maximum speed for the moving parts to be 40 feet/second and this implied, with proper carriage control, sixty additions or subtractions per minute (and that for 50 digits numbers) but only one multiplication or division in that time.

The first real implementation of Babbage's ideas came in 1937 when Howard Aiken planned an automatic electromechanical calculating machine. Working with IBM - if only Babbage had worked for the same firm - he produced the Automatic Sequence Controlled Calculator, controlled by paper tape input. Aiken in his reports makes explicit reference to Babbage, but, curiously enough, Aiken does not seem to have known about "judgement", and his Calculator did not have this facility. However, Aiken's requirements for a digital calculator were very similar to Babbage's, and perhaps rather more ambitious. The machine had to be able to do arithmetic, it must be able to recognize the sign of a number and the equality of two numbers, and it must also be capable of involution and evolution. In fact Aiken's "mill" was very similar to Babbage's. In going further, Aiken added sine and logarithm functions, all worked by plugboards, so there was an element of parallelism. The machines that followed used binary arithmetic rather than decimal, they used stored programs like modern computers, they were faster. Charles Babbage was the first man to consider seriously automatic computing implemented in terms of mechanism - what could he have done with the modern mechanism which enables one to put an "analytical engine" in a coat pocket?

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If he didn't say the things that he said to me that he did say, and his behaviour was not what he said that it was, the situation being entirely different, it is conceivable that a different conclusion would come from that material.

A psychiatrist testifying at the trial of Jack Ruby.

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To find a lucid geometric representation for your non-geometric problem could be an important step toward the solution.

How to Solve It, G. Polya, 1957.

HANNA NEUMANN

adapted from an obituary by M.F. Newman*

Hanna Neumann was born in Berlin on 12 February, 1914, the youngest of three children of Hermann and Katharina von Caemmerer. Her father was killed in the first days of the 1914-18 war and as a result the family lived impecuniously on a war pension which had to be supplemented by other earnings. Already at the age of thirteen Hanna contributed to the family income by coaching younger school children. By the time she reached the final years at school she was coaching up to fifteen periods a week. In 1922, after two years in a private school, she entered a girls' grammar school, from where she graduated early in 1932.

Her early hobby was botany. She collected plant specimens and built up voluminous herbaria for about four years until at about the age of fourteen this interest was superseded by her interest in mathematics.

Hanna entered the University of Berlin in 1932. Her first year was full of excitement. The lectures in Mathematics and Physics were delivered by leaders in each field. As well as these formal courses she took full advantage of the German tradition of attending lectures on a wide variety of topics. She listened to Kohler, one of the originators of Gestalt theory, on Psychology; to the well known Jesuit Guardini on Dante; and to Wolff, the leading academic lawyer in Germany, on Common Law (his popularity was such that he always had overflow audiences in the biggest lecture theatre in Berlin).

In this first year at university besides the excitement of study and the inevitable coaching there were, because lectures started early (8.00 a.m. and sometimes in summer 7.00 a.m.) and finished late, coffee breaks. Hanna soon found herself in a group of people, all senior to her - some already with doctorates - many of whom were later to make their mark in mathematical circles. It included her future husband, Bernhard Neumann, who later became Professor of Mathematics at the Australian National University.

The friendship between Hanna and Bernhard started in January 1933 and quickly blossomed into something special. In August 1933 Bernhard left for Cambridge in England; it had become clear that

*The full obituary appears in Vol.29, No.1 of *The Australian Mathematics Teacher* (1973), pp.1-22. We thank the editor of that journal, Mr John Veness, and Dr Newman for their permission to use this material in *Function*.

Germany would not be a place for Jewish people for some time to come. At Easter of 1934 Hanna visited Bernhard in London and they became secretly engaged; already the climate in Germany, and soon the law, was against such 'mixed' marriages. Then Hanna returned to her studies.

Her university course involved attendance at lecture courses, problem classes, practical classes, seminars and a physical education course (swimming in Hanna's case). There was also a final examination.

There is a story about the practical Physics class in her second year which illustrates a significant feature of Hanna's make-up. During the course the students, working in pairs, were required to use a theodolite to measure the height of a distant chimney stack. Hanna and her partner made the measurements, did the appropriate calculations and took the work for marking. They were told their result was significantly wrong and to repeat the work. This they did with essentially the same result. They were then told how far short their result was and to try again. They did with again much the same result. They then managed to persuade the demonstrator to check the measurements. Much to his surprise his agreed with theirs. Investigations revealed that a few years earlier the stack had been lowered by several courses of bricks!

For the fourth and final year of her degree Hanna chose to be examined in Mathematics, Physics and Philosophy. This involved an oral examination in all three subjects and extended essays in Mathematics and Philosophy. The summer semester of 1936 was spent preparing for the orals in August, but her preparation was seriously disrupted by an attack of scarlet fever. Nevertheless she obtained distinctions in both Mathematics and Physics and good marks in Philosophy, for an overall award with distinction.

During all this time Hanna and Bernhard kept in contact by correspondence. It was, in the circumstances, not an easy correspondence; it was conducted anonymously through various friendly channels. They met only once during this period - in Denmark for a couple of weeks in 1936 when Bernhard was travelling from the International Congress of Mathematicians in Oslo.

Following the completion of her first degree Hanna was accepted as a research student at the University in Göttingen, Germany. She also found a minor tutoring job with which she could finance her stay. Before taking up studies there in the summer of 1937, Hanna spent six months working in the statistics department of an institute of military economics.

In Göttingen Hanna found time for some chess and some gliding. She also found time to attend a course on Czech - this because a friend wanted to learn the language and the minimum class size was two. The course was no hardship as Hanna had a flair for learning languages, one that she put to good use later in her professional career in reading papers in a wide variety of languages.

Early 1938 saw the annexation of Austria and summer the Czechoslovak crisis. Hanna decided it would be impossible to complete her course without risking a prolonged delay in her marriage plans. So, after three semesters, she gave up her course and in July 1938 went to Britain.

The first years in Britain were far from easy. Yet they saw the beginning of her family, and the beginning of productive research. Hanna and Bernhard felt they could not openly marry until his parents were safe from possible reprisals. Bernhard was a Temporary Assistant Lecturer in Cardiff. Hanna went to live in Bristol.

Late in 1938 Hanna and Bernhard were secretly married in Cardiff. They finally set up house together in Cardiff early in 1939 when Bernhard's parents joined them. Later that year their first child, Irene, was born. During this time in Cardiff Hanna's earlier interest in botany was turned to practical use. The family were able to vary and supplement their diet with the use of such plants as sorrel which could be found growing wild.

Both Hanna and Bernhard were classified as 'least restricted' aliens. This meant that at first they were not affected by restrictions on aliens. However, after Dunkirk a larger part of the coast was barred to all aliens and they were required to leave Cardiff. They moved to Oxford - because it was a university town. Within a week Bernhard was interned and a few months later released into the British army. Meanwhile Hanna, expecting a second child, made arrangements to complete a doctorate (D.Phil.). This was made possible by the Society of Oxford Home Students (later St Anne's College) through which she enrolled, and a generous waiver of fees that Oxford University granted to all refugee students whose courses had been interrupted. Just after Christmas the second child, Peter, was born (he has become a mathematics don at Oxford after himself gaining a D. Phil. from there).

The major problem during this time was accommodation. The original flat became unavailable towards the end of 1941. It was not easy to find accommodation with two young children and was made no easier by having to compete with refugees from the bombing of London. All Hanna could find was a subletting of part of a house - with shared facilities. A year later another move became necessary. This time Hanna found a briiliant solution. She rented a caravan and got permission from a market gardener to park it on his farm. She also, as was necessary, had it declared 'approved rooms' by the Oxford Delegacy of Lodgings.

It was then that the thesis was largely written; in a caravan by candlelight. The typing was done on a card-table by a haystack when the weather permitted. The thesis was submitted in mid-1943. Soon after, restrictions on aliens were eased and Hanna was able to return to Cardiff. In November of that year the third child, Barbara, was born (she graduated in Mathematics from Sussex University and went on to teach mathematics in a secondary school). The oral examination took place in Oxford in April, 1944. Hanna returned to Cardiff with her D.Phil.

A year later the war in Europe was over. Bernhard was demobilized from the army and resumed his university career at the beginning of 1946 with a Temporary Lectureship at the University College in Hull. At the same time the fourth child, Walter, was born (after studying at universities in New York, Adelaide and Bonn, he gained a doctorate and is now active in mathematical research). For the next academic year (these run from October to June in Britain) Bernhard was made a Lecturer. Hanna was offered a Temporary Assistant Lectureship which she took and thus began her formal teaching career.

Hanna was to stay in Hull for twelve years rising through the ranks to be by the end of her time there a Senior Lecturer.

She took an active interest in her students. She was a strong supporter of the student mathematical society. She gave lectures to it on a number of occasions on such topics as: Dissection of rectangles into incongruent squares; Difficulties in defining the area of surfaces; and Prime numbers. Her aim was to exhibit some of the facets of mathematics for which there was not enough time in the regular courses and, as always, to convey her joy in mathematics. It was one of Hanna's striking qualities that she found joy in so much. The model building group also had her active support; in particular she participated in the making of paper models of regular and other solids. The outstanding feature, though, was her coffee evening. She often invited staff and students to meet at her house over coffee. This turned into a regular weekly open house at which her students were always welcome and, as one of her colleagues of those times says, "many benefited greatly from being able to drop in for company, discussion and often help with personal affairs". She was very interested in people and in seeing that they made the most of their abilities. One finds over and over that her interest in someone's work and her encouragement of it played a significant role.

Meanwhile the family thrived and grew with the addition of a fifth child, Daniel, born in 1951 (he has completed a university course in Mathematics and Greek and is now a violinist with the Melbourne Elizabethan Orchestra). This was, of course, a very busy time for Hanna. Even with a home-help (in whom she invariably inspired intense loyalty), she had to be well-organized and call on all her resources of stamina, will-power and self-discipline. Visitors were always struck by the organization of the children: all had tasks to do and carried them out with responsibility and efficiency.

From 1948 Bernhard had been lecturing at Manchester University and at various times from then on Hanna looked for a suitable position in Manchester so that the family could lead a life under one roof. This search finally succeeded in 1958.

At Manchester Hanna set about organising courses which would show the students something of mathematics as she saw it. She developed a style of teaching which aimed at making the acquisition of very abstract ideas accessible through judicious use of concrete examples and graded exercises. She tried to emphasise to the undergraduate students that parts of mathematics other than calculus were being applied to branches of human endeavour other than physics.

Hanna also set about building up an active teaching and research team around her.

Life thus continued very busy. Hanna would sometimes work all night reading manuscripts or preparing lectures, take a good long shower and appear in the office seemingly as fresh as if she had had a night's sleep. She did not allow this pressure of work to interfere with her contact with fellow staff and students nor with taking an interest in their work. There were regular coffee sessions at which they would discuss problems of interest. She was not beyond getting new experiences such as that of wall-papering.

One of Hanna's research interests was in an area of pure mathematics known as group theory. Group theory is studied by mathematicians largely for the fascination of its problems and the appeal of its ideas. However certain aspects of it have proved useful in the application of mathematics to various fields but especially physics. While Hanna was always at pains to stress that she saw the intrinsic motivations of beauty and joy as quite crucial, she was also interested in exploring such applications. Therefore she agreed to take part in a post-graduate course run by mathematicians and physicists on representations of groups. The mathematicians were to begin by giving a detailed account of those parts of the theory of interest to the physicists and then the physicists were to take over and explain how the theory was used. Hanna gave the mathematical lectures during 1960-1; the physical part never eventuated.

During 1960-1 preparations were made for a joint study leave by Hanna and Bernhard at the Courant Institute of Mathematical Sciences in New York in 1961-2; Hanna was a Visiting Research Scientist. It was also then that an offer came to Bernhard to set up a research department of mathematics at the Australian National University. Hanna was offered a post as Reader (now called Professorial Fellow) in that department. They accepted, with Bernhard to take up his appointment after the year in New York and Hanna a year later after discharging her obligations to her research students in Manchester.

In August 1963 Hanna left Britain to face new challenges in Australia. Hanna came to a research post in which she hoped to pursue her research interests and guide some research students to doctorates.

Instead Hanna found herself heading into major teaching responsibility. She was invited to take the newly created chair of Pure Mathematics in the National University's School of General Studies (that is the part of the university which is responsible for the teaching of undergraduate students and in which the academic staff are expected to devote a significant part of their time to teaching duties). With the chair went the headship of the Department of Pure Mathematics which, together with the Department of Applied Mathematics, had grown out of the fission of the former Department of Mathematics. She accepted the invitation and took up the appointment in April, 1964.

She also quickly became involved with helping teachers in secondary schools with some of the problems being created by the introduction of the Wyndham scheme into secondary schooling in

N.S.W. This scheme involved a radical restructuring which forced the creation of new syllabuses. In mathematics these new syllabuses reflected some of the changes that were taking place in the teaching of mathematics in other parts of the world. Many teachers found that their training had not prepared them to teach some aspects of these syllabuses. In the first term of 1964 Hanna and Ken Mattei, one of the mathematics masters in Canberra, ran (under the auspices of the Canberra Mathematical Association) a once-a-week course for teachers entitled "The language of sets in school mathematics". This was Hanna's first excursion into this kind of activity, however her experience and sensitivity enabled her to hit the right note and she was thanked "... for the lessons and guidance given so cheerfully and efficiently". This direct involvement with secondary teachers was to continue for the rest of her life.

Hanna was concerned to see that all students got courses suited to their needs. On the one hand she wanted the better students to get a real appreciation of mathematics so that they could sensibly decide whether they wanted to make a career within mathematics and be well prepared to do so. In this respect, besides making available an intensive course of study through lectures, she instituted forms of examining, especially take-home assignments, which encouraged more sustained use of the ideas and techniques involved than the conventional short closed-book examination. She also made a supervised project an important component of the final honours year. While this was not intended, these projects occasionally produced original research some of which has been published. On the other hand she was deeply concerned that students with a limited background who were intending only one year's study of mathematics at university should get as clear an understanding as possible of the nature of the subject because many of these people would be required to make some use of mathematics later in their lives. She was keen to get over the idea that doing and thinking about mathematics can be joyous human activities, though it needed effort to get the rewards. She conveyed this by her own obvious joy in the subject and her willingness to work hard. The success of the intensive course is easily measurable: at least a dozen students have gone on to complete doctorates in such widely scattered places as Cambridge, Edinburgh and Oxford in Great Britain, Chicago and Seattle in U.S.A. and Kingston in Canada as well as in Australia; mostly in mathematics but also in computing, physics and the history of science. These doctorates have been attained by graduates from the honours classes of 1965 to 1968 and represent about half the graduates from those classes.

Hanna took on an increasing number of responsibilities which reduced the time she had for research and research-related activities. In Who's Who Hanna's recreations were listed as cycling and photography: indeed it was a common sight to see Hanna and Bernhard cycle to and from their offices or to their lunch-time coffee in the city. They also developed a fondness for four-wheel travel and saw much of Australia, especially the back-blocks which so many city-dwellers never see. The photography, which had been a brief interest during student holidays, was revived by coming across some old photos that she had taken. This interest was combined with the old interest in botany to build up an impressive collection of photographs of flowers and trees of all sorts but especially of many varieties of acacia. The chase for these involved much use of four wheels. It also resulted in bodily damage, and at least one broken rib is directly attributable to a chase after an elusive acacia. Such ailments had no noticeable effect on her work, and even a leg in plaster could do no more than keep her away from classes for a week - she still prepared the lectures for a colleague to give.

In the eight years Hanna spent in Australia she made quite an impact on the Australian mathematical community.

In 1966 Hanna was elected to be one of the foundation Vice-Presidents of the Australian Association of Mathematics Teachers. A little later in the same year she was elected Vice-President of the Canberra Mathematical Association and in 1967-8 became its President. When the A.N.U.-A.A.M.T. National Summer School for talented high school students was started in 1969 she was an enthusiastic supporter of it and on two occasions gave lectures on geometry which proved very popular. In March, 1969, her academic excellence was given further recognition by her election to a Fellowship of the Australian Academy of Science (F.A.A.).

Hanna took a 12-month study leave break (with Bernhard) in August 1969. No sooner were they back than Hanna was invited to make a lecture tour of Canada under the Commonwealth Universities Interchange Scheme. This was arranged for the (Canadian) winter of 1971-2. At the end of October 1971 Hanna set off on her Canadian lecture tour. On the evening of the 12th of November, while visiting Carleton University, Ottawa, she felt ill, admitted herself to hospital and quickly went into a coma. She died on the 14th without regaining consciousness.

At work Hanna believed in making herself available: as far as formal commitments allowed, she was always in her office with the door open. She encouraged students to seek help with their difficulties and she was often to be seen explaining a point at her blackboard. She also found herself helping students with non-mathematical problems. Her impact here is best summed up by the following extract from a letter by two students published in the local paper just after her death:

"We will remember her not only as a mathematician; she was a friend who always had a sympathetic ear for any student, and was never too busy.

We will always miss her tremendous dedication and sincerity, and the friendliness of her presence."

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That straight thinking and logical expression does not come naturally, even to many highly educated people, was brought home to me just recently during a concert. The room was hot, and after the interval the players returned in shirt sleeves, without their coats. The leader bowed and said, 'Excuse us, ladies and gentlemen, for wearing our shirts'. ... [Students should] spontaneously shudder on hearing someone apologizing for wearing his shirt when he means to apologize for not wearing his coat!

Hanna Neumann.

RUSSIAN ARISTOCRAT ARITHMETIC

Neil Cameron, Monash University

On a recent visit to a remote Scottish manor house (near the Mull of Kintyre) I gave my usual after dinner party piece, showing how any two whole numbers can be multiplied using only the operations of adding, doubling and halving while casting off remainders. The technique is probably familiar to you, is variously attributed to Russian Peasants and Ancient Egyptians and is easily understood once you appreciate binary arithmetic.

For example, the following scheme gives the product of 56 and 21.

$$\begin{array}{ccccc} 56 & 112 & 224 & 448 & 896 \\ 21 & 10 & 5 & 2 & 1 \end{array}$$

and $56 + 224 + 896 = 1176$ $(= 21 \times 56)$.

The idea is to keep halving the smaller numbers (while casting off any remainders) until the number 1 is reached, at the same time doubling the larger numbers an equal number of times, identifying the larger numbers corresponding to those smaller numbers which are odd and adding the former together.

Given any real number z, the *integer part* $[z]$ of z is that unique integer n such that $n \leqslant z < n + 1$. Let x be a positive integer and define a finite sequence of integers as follows:

$$x^{(0)} = x$$

and $$x^{(n+1)} = \left[\tfrac{1}{2}x^{(n)}\right],$$

for integers n, $0 \leqslant n < k$ where $x^{(k)} = 1$.

For example, $21^{(0)} = 21$, $21^{(1)} = [21/2] = 10$, $21^{(2)} = 5$, $21^{(3)} = 2$, $21^{(4)} = 1$. Now identify those r, $0 \leqslant r \leqslant k$ for which $x^{(r)}$ is odd and call these values of r, $n_1, n_2, \ldots n_m$, where $0 \leqslant n_1 < \ldots < n_m = k$.

Then you can check that

$$2^{n_m} + \ldots + 2^{n_2} + 2^{n_1} = x$$

is the binary decomposition of x. For example, $2^4 + 2^2 + 2^0 = 21$ is the binary decomposition of 21.

We can now justify the above scheme. If y is the larger number and x the smaller number then

$$xy = (2^{n_m} + \ldots + 2^{n_2} + 2^{n_1})y$$
$$= 2^{n_m}y + \ldots + 2^{n_2}y + 2^{n_1}y,$$

the sum of the appropriate large numbers. For example, $21 \times 56 = 2^4 \times 56 + 2^2 \times 56 + 2^0 \times 56$.

"What about division?", asked my host since this is a much harder exercise for pupils at school. We all know that to do long division you must be able to subtract. It is not hard to find a scheme for division involving only the operations used before *together with* subtraction. The following is a scheme for finding the quotient of 1176 and 56, which, given the extra sophistication, I am frivolously calling Russian Aristocrat Arithmetic.

1176	588	294	147	73	36
896	448	224	112	$\boxed{56}$	
16	8	4	2	1	
280	140	70	35		
224	112	$\boxed{56}$			
4	2	1			
56					
$\boxed{56}$					
1					
0					

and 16 + 4 + 1 = 21 (= 1176/56).

How does it work? Why does it work? Perhaps a look at the following scheme for 1176/21 will help you discover.

1176	588	294	147	73	36	18
672	336	168	84	42	$\boxed{21}$	
32	16	8	4	2	1	
504	252	126	63	31	15	
336	168	84	42	$\boxed{21}$		
16	8	4	2	1		
168	84	42	21			
168	84	42	$\boxed{21}$			
8	4	2	1			
0						

and 32 + 16 + 8 = 56 (= 1176/21).

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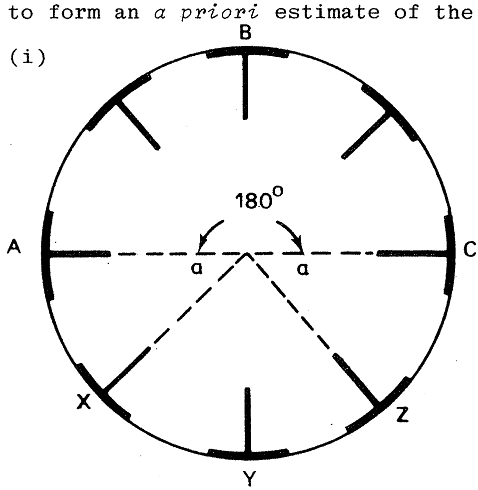
A MODEL FOR AN *A PRIORI* PROBABILITY

J.W. Hille, S.C.V. Frankston

When discussing probabilities of events, few people have any hesitation in accepting Prob{Head} = ½ when a coin is tossed or Prob{Six} = $\frac{1}{6}$ when a die is rolled. However, if a drawing pin is thrown onto a surface, it may land either with its point up or its point down, and in estimating Prob Point Down , many people have difficulty suggesting a value in which they have reasonable confidence. Questions of pointer length, mass and diameter of head etc. may prevent us from making a sound intuitive estimate, so I decided to analyse the possibilities of a pin reaching equilibrium to form an *a priori* estimate of the required probability.

(i)



Let the "circle of positions" shown ~~in Fig. 1~~ opposite, represent possible pin positions just prior to coming to rest. For positions *A* to *C* via *B* the pin should come to rest in the point down position (event *P*, say). Since this occurs for positions covering 180° of the possible 360° for the pin, this suggests Prob{*P*} is at least 0·5.

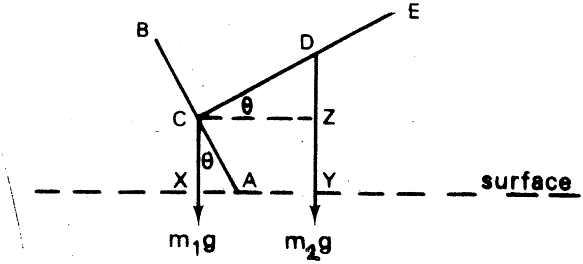
(ii) For positions *X* to *Z* via *Y*, the pin should come to rest head down (event *P*') and if α represents the "balance point" of the pin, the model suggests event *P* is realised if the pin occupies any positon between *X* and *Y* via *B* before coming to rest.

Assuming that all possible positions of the pin are equally likely we are led to predict that:

$$\text{Prob}\{P\} = \frac{180 + 2\alpha}{360} \quad (\alpha \text{ in degrees}).$$

(iii) α may be obtained by a consideration of the moments as follows:

Consider the vertical section through the pin (ABE) with head centre of mass at C and pointer centre of mass at D.



Let $CA = d_1$, $CD = d_2$, mass of head $= m_1$, mass of pointer $= m_2$ and angle $ACX = \theta =$ angle ZCD. The pin will return to the point down position if:

$$m_2g.AY > m_1g.AX \quad \text{(moments about } A\text{)}$$

i.e. $m_2.(CZ - AX) > m_1.AX$

i.e. $m_2.(d_2\cos\theta - d_1\sin\theta) > m_1.d_1 \sin\theta$

i.e. $\cot\theta > (d_1/d_2)(1 + m_1/m_2)$

i.e. $\tan\theta < \dfrac{1}{s(1+k)}$ where $s = d_1/d_2$, $k = m_1/m_2$.

Hence, $\alpha = \mathrm{Tan}^{-1}\{1/s(k+1)\}$.

Since $\tan\alpha$ (and hence α for the range involved) depends inversely on s and k, pins with large, massive heads give small α values and Prob$\{P\}$ will not be much in excess of 0·5. For pins with small, light heads, α may be quite large and Prob$\{P\}$ may be considerably in excess of 0·5. The table gives some selected values of s and k and the expected value of Prob$\{P\}$.

$s = 0{\cdot}5$

k	1	2	5	10	20	50
$\alpha = \mathrm{Tan}^{-1}\{1/s(k+1)\}$	45	33·7	18·4	10·3	5·4	2·2
Prob$\{P\}$	0·75	0·69	0·60	0·56	0·53	0·51

$s = 1{\cdot}0$

k	1	2	5	10	20	50
$\alpha = \mathrm{Tan}^{-1}\{1/s(k+1)\}$	26·6	18·4	9·5	5·2	2·7	1·1
Prob$\{P\}$	0·65	0·60	0·55	0·53	0·52	0·51

(iv) A test of the derived results then depends on knowledge of s and k and available pins were analysed as follows:

Treating the pin as circular disc with attached cylinder, a rough estimate of k was found from

$$\frac{m_1}{m_2} = \frac{\pi d_1^{\,2} t_1 \rho}{\pi (t_2/2)^2 \, 2d_2 \rho} = \frac{2d_1^{\,2} t_1}{t_2^{\,2} d_2} ,$$

where t_1 = thickness of head, t_2 = thickness of pointer, and ρ is the density.

Only three types of pins were available for measurement and these gave the following approximate results:

	Type I	Type II	Type III
d_1	5·5 mm	6·3 mm	4·8 mm
d_2	3·5 mm	4·0 mm	3·4 mm
t_1	0·8 mm	0·9 mm	0·6 mm
t_2	1·1 mm	1·35 mm	1·0 mm
s	1·57	1·58	1·41
k	11·43	9·80	8·13
Prob{P}	0·52	0·52	0·52

Only Type I pins were available in quantity and 100 pins were tossed 5 times giving 52, 52, 53, 62, and 52 cases of pins in the point down position. The observed proportion of $\frac{271}{500}$ = 0·54 provided some confirmation of the model's validity and I found it interesting to note that a pin appears to behave in a manner not very different from that of a "fair" coin.

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A CORRECTION AND AN APOLOGY

Although we try to keep errors out of *Function* as best we can, mistakes do creep in from time to time. Usually, these are unimportant, and we do not bother to draw attention to them. Regrettably, Volume 2, Number 5 contained two misprints of a rather more serious nature.

Our solution to Problem 1.1 on page 28 has (in its second line) 2^n being divisible by an odd number, which is, of course, a nonsense. The number 2^n should have been $2^n - 1$.

We also apologise to George *Strugnell*, author of the article on Martian calendars, for our mis-spelling of his name.

TOPICS IN THE HISTORY OF STATISTICAL THOUGHT AND PRACTICE

III. THE COMMUNICATION OF CHOLERA AND AN EXPERIMENT ON THE GRANDEST SCALE

Peter D. Finch, Monash University

Cholera is caused by a bacillus which is mainly spread by infected water though, like typhoid, it can also be spread by flies, infected food and carriers. Fever develops after a short incubation period; this is followed by abdominal pain, severe vomiting and diarrhoea. Loss of fluid leads to muscle cramps and the motions become of the so-called 'rice-water' type, i.e. there is no solid matter and the appearance is that of water to which a little milk has been added. This stage is followed by collapse and, if adequate treatment is not available, death results in about 70 per cent of cases.

Cholera was unknown to western medicine until about 1769 when it was encountered in India, many thousands dying in an epidemic which did not subside until 1790. From that time it gradually spread throughout Europe, America and Asia. It reached England in 1830 and the first case in London was recorded in the Autumn of 1848; this was followed by the epidemic of 1849. It was particularly virulent in the Crimea during the siege of Sevastopol 1854-5 when, in a period of only six months, 73 per cent of 8 regiments of the British army died from disease, principally cholera.

The way in which cholera is communicated was established by John Snow, M.D. (1813-58) in a work which has become one of the classics of epidemiology. Once cholera had reached England its control and prevention became a matter of urgency to the medical profession of that country. Various puzzling facts came to light and theories purporting to explain them were proposed. At that time modern bacteriology had yet to be developed and the causes and nature of disease were not understood. It had been discovered that some diseases could be artificially transmitted by inoculation of 'morbid matter' but this had not been demonstrated for the enteric infections and it was not even clear that the latter were, in fact, communicable.

After the London epidemic of 1849, Dr William Farr, one of the pioneers of medical statistics, noticed a remarkable coincidence between mortality from Cholera in different districts and their elevation, the higher districts suffering less than the lower ones. He suggested that the level of soil might have some direct influence on the prevalence of cholera. In 1850 a Mr John Lea, of Cincinnati U.S.A., advanced his 'geological' theory that the cholera-poison existed in the air about the sick but required calcareous or magnesian salts in the drinking-water to give it effect. He was led to this theory by noting that in the west of the U.S.A. cholera had attacked districts using calcerous water but passed by those using sandstone or soft water. Another view held that the disease was communicated by effluvia given off by the patient into the surrounding air and inhaled by others. Some supported the localisation theory which supposed that cholera was due to an unknown something in the atmosphere which became localised and had its effects increased by gases given off from decomposing animal and vegetable matters. None of these theories could explain all of the facts which gradually came to light. Snow argued that cholera was communicated from person to person by contamination of food and drink from the evacuations of those already affected, principally through contaminated water supplies. He showed how this theory accounted for all the known facts.

Some of the things to be explained were puzzling. In the first place there was evidence to suggest that cholera was not communicable. There were numerous instances of persons having contact with those affected and yet not themselves going down with cholera. A striking instance of non-communication, which was well-documented, occurred in 1814 when cholera appeared with great severity in the 1st bat. 9th regt. N.I.* on its march from Jaulnah to Trichinopoly. For another battalion, which accompanied it, did not suffer cholera, even though it had seemingly been exposed to exactly the same circumstances. Again a note had appeared in the Medical Times and Gazette for 1854 pointing out that, at the Newcastle dispensary the previous year, one of the dispensers drank by mistake some rice-water evacuation without ill-effect. Against this it was observed that the duration of cholera in a place was usually in direct proportion to the size of its population. It remained but two or three weeks in a village, two to three months in a good-sized town, whilst in a large city if often remained a whole year or longer. This pointed to the propagation of the disease from patient to patient for, as Snow remarked, '*...if each case were not connected with a previous one, ..., there is not reason why the twenty cases which occur in a village should not be distributed over as long a period as the twenty hundred cases which occur in a large town*'.

Another puzzling fact was that in England cholera always started in autumn, made little progress during winter and spring but increased rapidly to reach its climax at the end of summer; it then declined as the cooler weather set in; whereas, in Scotland, cholera ran its course right through winter immediately following its introduction in autumn. Again there seemed to be a curious sex effect. At the beginning of an epidemic male deaths exceeded those of females yet when the epidemic reached its peak that situation was reversed and deaths of females exceeded those of males. It was noted too that mortality from cholera varied considerably with occupation. For example, it was 1 in 24 for both sailors and ballast-heavers, 1 in 32 for coalporters and coalheavers but only 1 in 265 for medical men, 1 in 325 for undertakers and 1 in 1,572 in footmen and men servants. Moreover medical statistics showed that the mining population of Great Britain had suffered more from cholera than any other occupation.

Snow was able to explain all these facts. He started with detailed case studies from all over Britain and convincingly demonstrated both the communicability of cholera and its mode of communication. A typical case study concerned a man from Hull, where cholera was prevalent, who went to Pocklington, lodged with a Samuel Wride, was attacked by cholera on the day he arrived, September 8, and died the next day. Wride was attacked on

*Native Infantry

September 11 and died shortly afterwards. A neighbour, Mrs Kneeshaw, who had visited Wride at that time went down with cholera on September 9 and her son on the 10th. He died on September 15, she lived three weeks. On September 16, Mr and Mrs Flint, and Mr and Mrs Giles Kneeshaw, and two children, came from York to visit the sick Mrs Kneeshaw at Pocklington. There had been no cholera in York for some time. They all returned to York the next day except Mr G. Kneeshaw who stayed at Pocklington until September 24 when he returned to York. He was attacked by cholera on his return and died the next day. On September 27 Mrs Flint was attacked but recovered. Her sister attended her, was attacked on October 1 and died October 6. The infection now spread in York. A Mrs Hardcastle went down with cholera on October 3 and died the same day. Miss Agar residing with her died of cholera on October 7. Mr C. Agar visited Mrs Hardcastle on October 3, was attacked the next day and died October 6. A Miss Robinson who came to take care of the house after the deaths of Mrs Hardcastle and Miss Agar was attacked and died of cholera on October 11. Many detailed studies like this, involving a painstaking tracing of the course of the disease, provided strong evidence in favour of communicability. But they did not in themselves explain how it was communicated.

Snow investigated the mode of communication in two ways. Firstly he examined particular outbreaks of cholera and showed that they were indeed associated with contamination of water supplies. The most famous of these was the case of the Broad Street pump which was dramatised by Richard Gordon (of the 'Doctor' series fame, 'Doctor at Large' etc.,) in 'The Sleep of Life', a novel about the discovery of anaesthesia to which Snow made important contributions. Secondly he indicated how contamination of food and drink by the evacuations of those already affected could explain general facts like those mentioned earlier. A typical example of the case-study approach is provided by his investigation of the cholera outbreak of 1849 in Albion Terrace. This had a very high mortality and was particularly striking because there were no other cases in the immediate neighbourhood; the houses opposite to, behind and in the same line at each end of those affected remaining free from cholera. Snow found that the affected houses shared a common water supply different from that of neighbouring houses. It came from a spring in the road, was conducted by a drain to the back of the houses and flowed into supply tanks for each of them. The tanks were placed on the same level and pumping from one drew water from the others so that any impurity getting into one tank was imparted to the rest. Under the privy of each house there was a cesspool situated near its water tank. On July 26 there was a storm, some of the overflow-pipes from the cesspools became blocked, burst and subsequently contaminated the water tanks. The first case of cholera occurred on July 28 and that person died on the same day. There were 2 deaths on August 3rd, 4 on the 4th, 2 on the 6th, 2 on the 7th, 4 on the 8th, 3 on the 9th, 1 on the 11th and 1 on the 13th. In addition to these 20 deaths there were some 4 or 5 more from those who fled the houses after the outbreak had started. Snow commented, *"There are no data for showing how the disease was communicated to the first patient, at No.13, on July 28th; but it was two or three days afterwards, when the evacuations from this patient must have entered the drains having a communication with the water supplied to all the houses, that other persons were attacked, and in two days more the disease* *prevailed to an alarming extent ... the only special and peculiar cause connected with the great calamity ... was the state of the water, which was followed by the cholera in almost every house to which it extended, whilst all the surrounding houses were quite free from the disease."*

Numerous examples like this seemed to confirm that cholera was indeed transmitted in the way Snow suggested, at least in special cases. But that type of argument, though important, could not go beyond the special case and it would scarcely be convincing to explain the widespread London epidemic of 1849, for example, by a succession of coincidences involving burst pipes and a large number of different contaminations of separate water supplies. Moreover it remained to be shown how the suggested mode of communication could explain the general phenomena noted earlier. Let us now indicate some of Snow's explanations of those phenomena.

The case of the two battalions, one infected the other not, was easily disposed of. The official report on that outbreak had noted that at Cunnatore the force had been so encamped, that while the 5th Native Infantry had their water supplied from wells, the 9th Native Infantry procured their water from tanks in low ground. Moreover it seemed that sepoys of low caste and camp followers had indiscriminantly bathed in those tanks. The inference is clear. Snow explained the disparity between the English and Scottish experiences in the following way. In England, he argued, people seldom drink unboiled water, except in hot weather. They generally drink tea or beer. In Scotland, however, beer and tea were not widely drunk and unboiled water was freely mixed with whiskey.

The changing relative mortality of the sexes was explained by noting that the greater part of the female population remained at home, whereas the men moved about in following their occupations and so experienced a greater initial risk. Later, when the cholera had spread, the women were equally at risk with the men but also endured the additional risk associated with nursing the sick. Disparity between the mortality rates of different occupations were explained in a similar way. Snow noted that those with a high mortality, like sailors, ballast-heavers, coalporters and coalheavers, all lived or worked on the river Thames, where it was the habit to drink water drawn by pailfuls from the side of a ship. Their higher mortality was to be expected because sewage was generally disposed of in the river. The high incidence of cholera among coal miners was explained when Snow learnt in response to enquiries that 'the pit is one huge privy'. He argued, *'There are no privies in the coal-pits ... . The workmen stay so long ... they are obliged to take a supply of food with them, which they eat invariably with unwashed hands, and without knife and fork.'* Farr's suggestion about the level of the soil was disposed of by citing instances of cholera in the most elevated towns of the country. Snow argued that the increased prevalence of cholera in the low-lying districts of London was related to the greater contamination of their water supplies. Finally he pointed out that the cited instance of a dispenser drinking rice-water evacuation without ill-effect was not necessarily opposed to his theory. For, he argued, though the cholera-poison is in those evacuations it is not necessarily uniformly distributed throughout it and could have been absent from the portion drunk.

A lesser man than Snow might well have stopped at this point. He had painstakingly collected a vast amount of data pertaining to the spread of cholera and he had produced a plausible 'theory' to account for it. The fact that he did not stop there is an object lesson for the social and educational theorists of today, some of whom seem to take the view that the mere expression of their opinion is sufficient grounds for the adoption of their suggested remedies for the ills of society. Snow remarked, "*I had no reason to doubt the correctness of the conclusions I had drawn from the great number of facts already in my possession, but I felt that the circumstances of the cholera-poison passing down the sewers into a great river, and being distributed through miles of pipes, and yet producing its specific effects, was so startling a nature, and so vast importance to the community, that it could not be too rigidly examined, or established on too firm a basis.*" Snow now sought to explain how the London cholera epidemic of 1849 had been spread by means of the water supply.

At that time water was supplied to houses in South London mainly by two private companies. The water was obtained from various sources, but much came from the Thames - that of one company from above, that of the other from below, the points at which sewage was discharged into it. Moreover the various companies often supplied the same areas, even different houses in the same street. Snow remarks:

"*Each company supplies both rich and poor, both large houses and small; there is no difference in either the condition or occupation of the persons receiving the water of the different Companies . ... As there is no difference whatever, either in the houses or the people receiving the supply of the two Water Companies, or in any of the physical conditions with which they are surrounded, it is obvious that no experiment could have been devised which would more thoroughly test the effect of water supply on the progress of cholera than this, which circumstances placed ready made before the observer.*

The experiment too was on the grandest scale. No fewer than three hundred thousand people of both sexes, of every age and occupation, and of every rank and station, from gentlefolks down to the very poor, were divided into two groups without their choice, and, in most cases, without their knowledge; one group being supplied with water containing the sewage of London, and amongst it, whatever might have come from the cholera patients, the other group having water quite free from such impurity.

To turn this grand experiment to account, all that was required was to learn the supply of water to each individual house where a fatal attack of cholera might occur..."

Snow proceeded therefore to the huge task of investigating cholera mortality as related to the two principal water supplies in South London, that contaminated by sewage from the Southwark and Vauxhall Company and the relatively pure water from the Lambeth Company. He obtained information on the total number of houses supplied by each company but he had no exact information about the number of houses supplied in each sub-district. While he was able to ascertain the number of deaths from cholera in the consumers of each water supply in each sub-district and to show that quite uniformly the number of deaths was greater among the consumers of the Southwark and Vauxhall than of the Lambeth supply, he was not able to calculate actual mortality rates in the two groups by sub-districts. This was done later by the General Board of Health which undertook an official enquiry and supplied the detailed statistics Snow had lacked. These confirmed Snow's conclusions. In the table below we reproduce some of Snow's results. It indicates that the death rate from cholera for those supplied with water by the Southwark and Vauxhall Company was almost six times as great as it was for those with water from the Lambeth Company; the more detailed results of the later official enquiry showed that it was, in fact, almost seven times as great.

	Population in 1851	Deaths by Cholera in 14 weeks ending Oct.14	Deaths in 10,000 living
London	2,362,236	10,367	43
West Districts	376,427	1,992	53
North Districts	490,396	735	14
Central Districts	393,256	612	15
East Districts	485,522	1,461	30
South Districts	616,635	5,567	90
Houses supplied by Southwark and Vauxhall Company	266,516	4,093	153
Houses supplied by Lambeth Company	173,748	461	26

Reference

Snow on Cholera: being a reprint of two papers by John Snow M.D. Hafner Publishing Company. New York and London. 1965.

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She saw every relationship as a pair of intersecting circles. It would seem at first glance that the more they overlapped the better the relationship: but this is not so. Beyond a certain point the law of diminishing returns sets in, and there are not enough private resources left on either side to enrich the life that is shared. Probably perfection is reached when the area of the two outer crescents, added together, is exactly equal to that of the leaf-shaped piece in the middle. On paper there must be some neat mathematical formula for arriving at this; in life, none.

Mrs Miniver, Jan Struther, 1939.

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Nothing is more expected than the unexpected.

Epilogues, Remy de Gourmont, 1906.

PROBLEM SECTION

Each issue of *Function* contains a set of problems for you to try. Please send us problems of your own and solutions to the problems we print. We give first some solutions to earlier problems.

SOLUTION TO PROBLEM 1.2.7

This problem drew attention to a remarkable approximation to sin x. The formula $S(x) = x(1-0\cdot16605x^2 + 0\cdot00761x^4)$ gives an approximation to sin x that is valid for all x in the domain $0 \leqslant x \leqslant \frac{\pi}{2}$. The maximum error is less than 2×10^{-4}.

SOLUTION TO PROBLEM 1.5.1

This problem asked which of a hoop, a disc and a sphere, each of radius R, and each rolled down an inclined plane, reached the bottom in the least time. By solving the equations of motion, we can show that the accelerations of the three are respectively $\frac{1}{2}g \sin \alpha$, $\frac{2}{3}g \sin \alpha$, $\frac{5}{7}g \sin \alpha$, where α is the inclination of the plant and g is the acceleration due to gravity.

However, a more elegant solution is possible. In each case, an initial potential energy is converted to a final kinetic energy which has two components - translational and rotational. The rotational component is least where most of the mass is concentrated near the centre - i.e. in the case of the sphere. The sphere thus has the highest translational kinetic energy at any time. Similarly, the hoop will have the lowest.

SOLUTION TO PROBLEM 2.2.4

The problem asked was:

Let n be an integer greater than 2. Prove that the n-th power of the length of the hypotenuse of a right angled triangle is greater than the sum of the n-th powers of the lengths of the other two sides.

The following elegant solution was supplied by Geoffrey Chappell, then a student at Kepnoch High School, Bundaberg:

Let x,y be the sides and z the hypotenuse: $x,y,z > 0$ and $x^2 + y^2 = z^2$. Define h by $y^2 = hx^2$ so $h > 0$.

$$z^2 = (h + 1)x^2, \quad x^n + y^n = (h^{n/2} + 1)x^n,$$

$$z^n = (z^2)^{n/2} = (h + 1)^{n/2}\, x^n.$$

Now $z^n - (x^n + y^n) = [(h + 1)^p - (h^p + 1)].x^n$

where $p = \frac{n}{2} > 1$.

$$z^n - (x^n + y^n) = g(h).x^n, \quad (h > 0) \qquad (1)$$

where

$$g(h) = (h + 1)^p - (h^p + 1)$$

$$g'(h) = p(h + 1)^{p-1} - p.h^{p-1} = p[(h + 1)^{p-1} - h^{p-1}].$$

But because $p - 1 > 0$, then, for positive h, h^{p-1} is an increasing function of h and $(h + 1)^{p-1} - h^{p-1} > 0$. Now $p > 0$ so for $h > 0$, $g'(h) > 0$ or $g'(h) > 0$. This implies that $g(h) > g(0)$. But $g(0) = 0$ so $g(h) > 0$. Referring to (1), $x^n > 0$ because $x > 0$; $g(h) > 0$ so $z^n - (x^n + y^n) > 0$ or $z^n > x^n + y^n$.

SOLUTION TO PROBLEM 2.4.1

The problem was to simplify the statement:

If Monday is a public holiday, then I will not go to the beach, or I will stay at home, or I will neither stay at home nor go to the beach.

Assuming that the speaker's home is not on the beach, we can simplify his statement to this: "If Monday is a public holiday, I will not go to the beach."

SOLUTION TO PROBLEM 2.4.2

There are 700 hymns in a church hymn book. It is required to print a set of cards, each with one digit on it, so that the numbers of any four hymns (to be sung on Sunday) can be displayed on a notice board. How many cards are required? (Give two answers, one assuming that an inverted 6 can be used as a 9, the other without that option.)

First consider the case without the option. The hymns could be 111, 121, 131, 141 or some other combination needing nine 1's. Similarly nine cards are required for each digit 2,3,4,5,6. In the case of 0, we could have 100, 200, 300, 400 or some such. We require eight 0's. Similarly eight cards are required for each digit 7,8,9. The total is 86.

Where we do have the option, no nines are needed as such, but we could have 696, 669, 666, 699, for example. We save eight nines but require three extra 6's. The total is 81.

SOLUTION TO PROBLEM 2.4.3

Readers were asked to find the number of 0's at the end of 1000!, and were referred to Derek Holton's article in *Function*, Vol.2, No.4. Following the reasoning of Phase 4 of the article, we find that the number is $\frac{1000}{5} + \frac{1000}{25} + \frac{1000}{125}$, i.e. 248.

SOLUTION TO PROBLEM 2.4.4

From the roof of a 300 metre building in New York, two marbles are dropped, one being released when the other has already fallen 1 mm. How far apart will they be when the first hits the ground?

Let g be the acceleration due to gravity, S the height of the building, ΔS the initial discrepancy, $S - S'$ the final discrepancy. The first marble takes time T to fall where $T = \sqrt{2S/g}$. The initial time-discrepancy is $\Delta T = \sqrt{2\Delta S/g}$. When the first marble hits the ground the second has been in flight for a time $T - \Delta T$, and has travelled a distance $S' = \frac{1}{2}g[\sqrt{2S/g} - \sqrt{2\Delta S/g}]^2$ i.e. $S' = S + \Delta S - 2\sqrt{S\Delta S} \simeq S - 2\sqrt{S\Delta S}$ as ΔS is small. Then $S - S' = 2\sqrt{S\Delta S}$ to high accuracy. For the figures given, this works out to be about 1.1m.

We will not yet give the solution to

PROBLEM 2.3.2.

What point on the earth's surface is furthest from the earth's centre?

No correct answer to this interesting (and non-trivial) problem has yet reached the editors.

Here are some new problems.

PROBLEM 3.1.1

Two of a 3-man jury each independently arrive at a correct decision with probability p. The third flips a coin. The decision of the majority is final. What is the probability of the jury's reaching a correct decision?

PROBLEM 3.1.2

The product of four consecutive integers is a square - find the integers. Do the same for the case of four consecutive odd integers.

PROBLEM 3.1.3

In the diagram opposite, the lines drawn are the set of all common tangents to the circles. Prove that the points A, B, C are collinear.

PROBLEM 3.1.4

A man and a horse run a race, one hundred metres straight, and return. The horse leaps 3 metres at each stride and the man only 2, but then the man makes three strides to each two of the horse. Who wins the race?

PROBLEM 3.1.5

In a tennis tournament there are $2n$ participants. In the first round of the tournament each participant plays just once, so there are n games each occupying a pair of players. Show that the pairings for the first round can be arranged in exactly

$$1 \times 3 \times 5 \times \ldots \times (2n - 1)$$

different ways.

PROBLEM 3.1.6

Hanging over a pulley is a rope with a weight at one end. At the other, there is a monkey of equal weight. The rope weighs 250 gm per metre. The combined ages of the monkey and its father total 4 years and the weight of the monkey is as many kilograms as his father is years old. The father is twice as old as the monkey was when the father was half as old as the monkey will be when the monkey is three times as old as the father was when he was three times as old as the monkey was. The weight of the weight plus the weight of the rope is half as much again as the difference between the weight of the weight and the weight of the weight plus the weight of the monkey.

How long is the rope?

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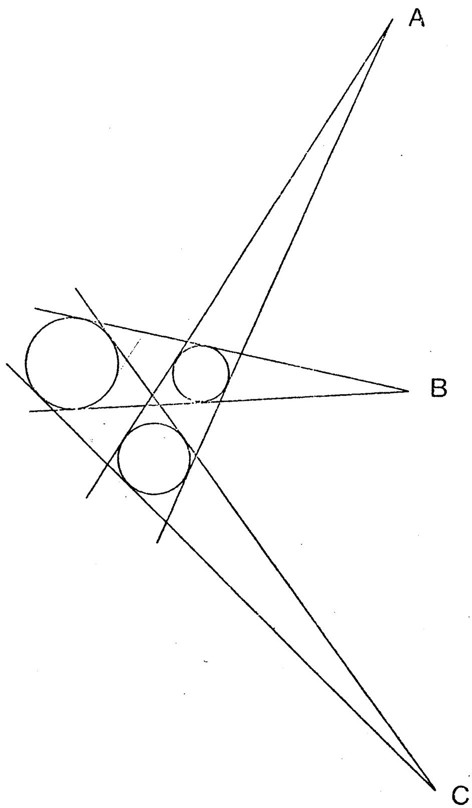

A BRITISH DEVELOPMENT

The London Observer for 3.12.78 carried an item by Auriol Stevens, their education correspondent. Its heading was "Sixth form maths may be made compulsory". It read in part:

"Mathematics will become compulsory in some sixth forms if suggestions put forward by a working party of independent school headmasters last week are adopted.

Maths, the headmasters say, has become essential not only for scientists and technologists but for any educated person. They have designed a one-year course for non-mathematicians, and are asking members of the Headmaster's Conference to experiment with it

The move reflects the growing importance of mathematics. Demand for courses by schoolchildren and for maths qualifications by employers, universities, polytechnics and colleges has far outstripped supply. Schools are having difficulty staffing the classes needed. Maths graduates are able to take their pick of jobs.

About 3 400 maths graduates are produced each year, and, although the number of maths and science graduates teaching in schools has trebled in the last 25 years, figures published by the Education Deaprtment show a sharp rise in the demand for maths teachers, with more than 1 100 vacant places."

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MONASH LECTURES ON MATHEMATICAL TOPICS, 1979

Monash University Mathematics Department invites secondary school students studying mathematics, particularly those in years 11 and 12 (H.S.C.), to a series of lectures on mathematical topics The first lecture of 1979 will be given by the Chairman of the Mathematics Department, Professor G.B. Preston, who will speak on

"Mathematical Paradoxes",
Friday, 23rd March, 1979,
at 7 p.m. in the Rotunda lecture theatre R1.

The lectures are free, and open to teachers and parents accompanying students. Each lecture will last for approximately one hour, and will not assume attendance at other lectures in the series. The lecture theatre R1 is located in the "Rotunda", which is linked to the Alexander Theatre at Monash. For further directions please enquire at the Gatehouse, in the main entrance in Wellingto Road. Parking is possible in any car park at Monash.

Further lectures are planned for April 6, 20; May 4; June 8, 22; July 6, 20; August 3, and details of these will be available at the first lecture and a complete program will also be posted to schools.

Enquiries: G. Watterson
Phone: 541 2550.

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At the time when I was new in Africa a shy young Swedish dairy-man was to teach me the numbers in Swaheli. As the Swaheli word for nine, to Swedish ears, has a dubious ring[+], he did not like to tell it to me, and when he had counted: 'seven, eight', he stopped, looked away, and said: 'They have not got nine in Swaheli.'

'You mean,' I said, 'that they can only count as far as eight?'

'Oh, no,' he said quickly. 'They have got ten, eleven, twelve, and so on. But they have not got nine.' .....

The idea of this system for a long time gave me much to think of, and for some reason a great pleasure. Here, I thought, was a people who have got originality of mind, and courage to break with the pedantry of the numeral series.

One, two and three are the only three sequential prime numbers, I thought, so may eight and ten be the only sequential even numbers. People might try to prove the existence of the number of nine by arguing that it should be possible to multiply the number of three with itself. But why should it be so? If the number of two has got no square root, the number of three may just as well be without a square number.

Out of ***Africa***, Karen Blixen, 1937.

[+] The Swahili (modern spelling) word for nine is *tisa*, which, in Swedish, means 'to piss'.

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I am rather surprised that you expect clarity from a professional philosopher. Muddled concepts and definitions are nowhere more at home than among philosophers who are not mathematicians ... . Just look around at today's philosophers, Schelling, Hegel, Nees von Esenbeck and Co. Don't their definitions make your hair stand on end? Or, in classical philosophy, read the kind of thing which 'stars' like Plato and others (I except Aristotle) gave as explanations. Even Kant is often not much better: his distinction between analytic and synthetic propositions is, I believe, one of those which either turns on a triviality or is false.

From a letter by C.F. Gauss to the astronomer Schumacher, dated 1.11.1844.

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"I'm sure of it", said he, "it's a round square in the shape of an oblong...". ...I found it as he said, the 'square' being really a triangle.

The Ripening Rubies, Max Pemberton, 1894.

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