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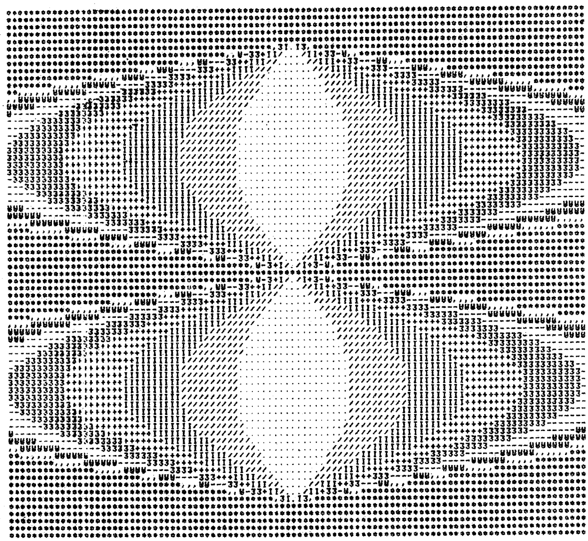

A SCHOOL MATHEMATICS MAGAZINE

Published by Monash University

Function is a mathematics magazine addressed principally to students in the upper forms of schools. Today mathematics is used in most of the sciences, physical, biological and social, in business management, in engineering. There are few human endeavours, from weather prediction to siting of traffic lights, that do not involve mathematics. *Function* will contain articles describing some of these uses of mathematics. It will also have articles, for entertainment and instruction, about mathematics and its history. There will be a problem section with solutions invited.

It is hoped that the student readers of *Function* will contribute material for publication. Articles, ideas, cartoons, comments, criticisms, advice are earnestly sought. Please send to the editors your views about what can be done to make *Function* more interesting for you.

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Articles, correspondence, problems (with or without solutions) and other material for publication are invited. Address them to:

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The magazine will be published five times a year in February, April, June, August, October. Price for five issues (including postage): $3.50; single issues 90 cents. Payments should be sent to the business manager at the above address: cheques and money orders should be made payable to Monash University. Enquiries about advertising should be directed to the business manager.

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We thank the Editor of *The Age* for permission to reproduce one of their letters on page 9. We thank Geoffrey Chappell for proving that part (i) of Problem 2.5 was wrong (see page 27). We thank John Taylor for permission to reproduce his article from *Student Mathematics*. *Student Mathematics* is a Canadian magazine for schools, published once a year. The address to write to if you want to buy it is at the foot of page 6. We thank Michael Eliott for the front cover picture which he devised and made and also for two other pictures of his reproduced on page 2.

We have received an increasing amount of correspondence and we hope the increase continues. Let us have contributions. Let us know subjects on which you would like articles.

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THE FRONT COVER

by J.O. Murphy, Monash University

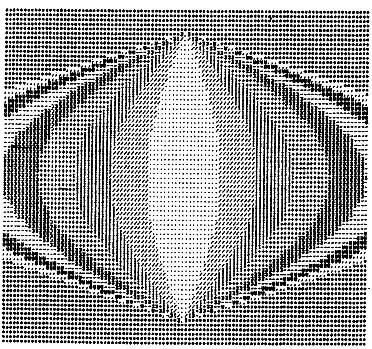
The computer generated design printed on the front cover of this issue of *Function* was produced by Michael Eliott, a fourth form student at Scotch College, Melbourne. Elements from the character set {*./+-,IW3} have been used to produce this graphic output on the line printer using A type format. The basic geometric shape, in this case part of the curve $x = 10\sin(\frac{y}{10})$, is determined by the statement line labelled 0010 (Z=SIN(Y/10.)*10.) in the FORTRAN source program reproduced on page 31.

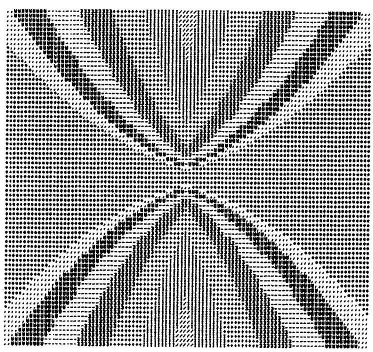
Once the geometry of the design has been decided the pattern is first established in the second quadrant and then extended to the other three quadrants by reflection about the coordinate axes. On the y axis one line represents one unit, and one character (note the characters are equally spaced) represents one unit on the x axis. Within each quadrant the different shading on a particular horizontal line (lines are printed only at integral values of y, as stated above) is obtained from the expression

$$x \text{ coordinate} = [p \times f(y)],\ 1 \leqslant p \leqslant 8,$$

each value of p identifying a member of the character set defined by the array $L(I)$ (see the program), [ ] meaning integral value of, and $f(y)$ standing for $10\sin(\frac{y}{10})$. Any remaining locations on the line are then filled by the * character.

Some other possible designs are illustrated below.





MATHEMATICS and LAW

by Sir Richard Eggleston

Monash University

Anyone who has to do with the law needs a knowledge of ordinary mathematical procedures. A jury of four in a Victorian country town once gave a very generous verdict, and the solicitor for one of the parties, being naturally curious as to how it was arrived at, took the opportunity of asking one of the jurors how they arrived at the verdict. He said they had found it difficult, but they had finally decided that each should write down on a piece of paper the amount that he thought should be awarded. Then he said, "We added them up ... My God, we forgot to divide by four!"

The earliest case, so far as I know, in which a mathematical formula was of significance was in 1663. A man had agreed to buy a horse, the price being calculated as follows: one barleycorn for the first nail in his shoes, two for the second, four for the third, and so on for the 32 nails. When he discovered that he was going to have to provide five hundred quarters of barley (How much was barley in 1663? Wheat was 36s. a quarter in 1780 when the first official records of prices were kept in Britain.) he tried to get out of his contract. The Court took a kindly view, the trial judge directing the jury to give the plaintiff the value of the horse, which they assessed at £8.

Forty years later in 1705, another defendant was in Court, having fallen for a similar trick. This time he had agreed to pay the plaintiff, in return for half a crown down, and a further £4.17.6 to be paid on completion of the contract, to deliver to the plaintiff a grain of rye on the following Monday, two grains the next Monday and so on for a year, doubling up each week. The defendant's counsel tried to argue that the contract was impossible to perform, as there was not so much rye in all the world. Chief Justice Holt pointed out that the contract as pleaded only required delivery every second Monday, which would presumably have required something less than the 500 qrs. of barley in the earlier case. Actually, this was probably said tongue in cheek. In those days the parties had to use Latin for their written pleadings, and the plaintiff's counsel used the phrase "quolibet alio die Lunae" which literally translated means "every other Monday", but the Latin phrase would not have been intended to have the same meaning as in colloquial English. In the end the defendant, seeing that he was not getting very far, offered to pay the half crown and the plaintiff's costs, and this was accepted. Counsel during argument in that case gave as an example of a contract that is obviously impossible of performance: "To go to Rome in a day". In more recent times "to make a journey to the moon" has been chosen as an example, but this too has now to be discarded.

Geometrical progressions play quite a part in legal problems. The most familiar is the calculation of compound interest and its variants. Most people are familiar with the basic idea that money lent at compound interest mounts up much faster than at simple interest. It doubles in 14 years at 5%, in 10 years at 7% and in 7 years at 10%. In each case it is a little longer, but these are good working figures to keep in one's head. Sometimes the question is as to the value of a reversion, that is to say a right to property of which the owner can only obtain possession on the death of a life tenant. Suppose a testator leaves a sum of $10,000 on trust to pay the income to A during his life and on his death to pay the capital to B. What is the present value of B's interest? This depends on what rate of interest the fund might be expected to earn, and on how long A is likely to live. If A's expectation of life is 14 years and the rate of interest is 5%, the present value of B's interest will be approximately $5,000 since $5,000 invested now at 5% compound would amount to $10,000 in fourteen years. In other cases a testator has directed his estate to be sold and the income given to one person, and the reversion to another. What do you do if a valuable work of art has to be retained for some years before a buyer has to be found? More important, how do you assess the value to an injured plaintiff of the loss of his earnings for the rest of his working life? In such a case you are really taking the present value of each of a series of payments, and adding them all together. There are of course tables available to save the lengthy calculations involved, but the neglect of the principle itself may have unexpected consequences. In the 1830's there was a wave of law reform in Britain. One step was to abolish some of the sinecures which were in the gift of the Lord Chancellor. As a first step provision was made for the Lord Chancellor to receive a pension of £5,000 a year, without which the reforms had no chance of success. Then it was provided that the officials of the Six Clerks' Office (including also the Sixty Clerks and the Waiting Clerks) should each be paid the net annual value of their fees and emoluments. This was taken to mean that each could have a lump sum equal to the annual value multiplied by the number of years of his expectation of life. The annual amount was to be the average of the last three years - greatly inflated by the recent efforts of two Vice-Chancellors to reduce arrears of work in the Court of Chancery. The top scorer received £214,768.9.10, and the next £163,575.11.10. The total for 29 persons was £1,358,424.3.5½. As few of these people did any work except to meet regularly for dinner and to divide up the takings, this must be regarded as having been a very generous settlement.

Reference to expectations of life leads to the mention of probabilities, and actuarial calculations. Of course, the mathematics of probability can become very complicated, and it is unwise for a lawyer to try to work these things out for himself, but he should be able to recognise the existence of the problem. Probability questions arise in many branches of of the law, but not least in the assessment of evidence. Fingerprint evidence, for example, depends on the probability of two persons having the same combinations of characteristics in their prints. As Sir Leo Cussen pointed out in 1912, an expert cannot say that no two persons can have the same fingerprints. He can only say that it is highly improbable. But the degree of improbability is very high, if you have a complete print. Galton put the odds against a random selection of a print corresponding with a given print as one in 64 000 000 000. But of course, the fingerprint people cannot often get a perfect print at the scene of the crime, and in such cases the odds may be very much less. Moreover, these odds relate to a random selection. If a matching print is found as a result of an extensive search the odds may be very much shorter.

A very interesting case on probabilities was the taxation case referred to by Dr G.A. Watterson in the February 1977 issue of *Function* [p. 7], in which the probability of two sets of bonds matching in all their known characteristics was in question. Incidentally, that case was of a kind that demands a particular kind of mathematical technique. "Betterment" cases (as they are called) are based on the idea that if a man cannot tell you how much he has earned, you can find out by making a list of his assets at the beginning and the end of the year, and adding on to any increase the amount he is assumed to have spent during the year for living expenses, or for any other item which will not be reflected in an increase in his assets. Put that way, it sounds simple, but in fact it is much more complicated. Capital gains have to be deducted and capital losses added on. Legacies and gifts to the taxpayer have to be deducted. The result is a fiendishly complicated experience in which you feel that you are doing all your arithmetic standing on your head. For my part, one betterment case in a lifetime is enough.

One could go on for some time listing the kinds of cases in which a knowledge of mathematics is a help. The industrial field, and now Trade Practices cases and cases before the Prices Justification Tribunal, involve an ability to work with statistics, calculations of movements in costs and profit margins and the like. In the 40-hour Week Case, an engineer employed by a public utility said that if working hours were reduced from 44 to 40 his labour costs would increase by 10%, and material costs by 6%, a total cost increase of 16%! The fallacy in this case is obvious, but a lawyer has to be prepared to expose more subtle fallacies, especially in dealing with statistics. The growing use of computers is going to involve lawyers in some understanding of the way in which computers operate. The increasing importance of a knowledge of company accounting need hardly be stressed.

Unfortunately, our education system tends to draw off students with a mathematical bent at an early age. If they show any promise in this field, they are encouraged to pursue scientific subjects, and by the time they reach Higher School Certificate level they are irrevocably committed to courses which make it difficult for them to shift back to the humanities. In the result, the best law students are often students of great ability in the humanities but no mathematical aptitude whatever. I do not mean to say that in order to cope with the kind of problem I have been discussing, it is necessary to have been trained to a high level in mathematical theory. It is however, an immense advantage to be able to grasp mathematical concepts, and above all, not to be frightened by them.

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A MATHEMATICAL EQUATION IN LEGAL LANGUAGE

From the Income Tax Assessment Act Section 67 (1):

Subject to this section, so much of the expenditure incurred by a taxpayer in borrowing money used by him for the purpose of producing assessable income as bears to the whole of that expenditure the same proportion as the part of the period for which the money was borrowed that is in the year of income bears to the whole of that period shall be an allowable deduction.

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PROBLEM 3.1.

Three men go on a fishing expedition. On the first day they catch a certain amount of fish and without cooking any - being tired - they all camp down for the night.

During the night one man awakes and decides to go home. Without waking the others he makes 3 equal shares of the fish to take his share. However he finds 2 fish left over so he puts these 2 together with his share and takes them home.

A little while later another awakes and also decides to go home. He too makes 3 shares of what is left, finds 2 left over, takes these and his share and leaves.

The last man to go also makes 3 shares, finds 2 left over, takes these 2 and his share of fish and leaves.

How many fish did they catch?

Generalize this problem so as to answer the same question but now for M men, with a remainder of $N (0 \leq N < M)$ fish left over when each man in turn takes his share.

Submitted by Michael Moses, Science II, Monash

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Student Mathematics, Room 373, College of Education, 371 Bloor Street West, Toronto 181, Ontario, Canada. (See page 1.)

SUMS OF SQUARES†

by John Taylor
University of British Columbia

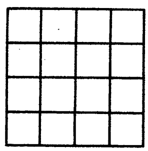

Figure 1

Given Figure 1 I was asked, "How many squares?" Like most people I answered, "Sixteen, - no there're more!" Further counting yielded $1^2 + 2^2 + 3^2 + 4^2$ squares altogether (do you agree?). I answered the same question for a 5 by 5 square (this gave $1^2 + 2^2 + 3^2 + 4^2 + 5^2$) and for a 3 by 3 square (this gave $1^2 + 2^2 + 3^2$), so it became apparent that there are $1^2 + 2^2 + 3^2 + \cdots + n^2$ squares in an n by n square of this type.

I decided to use this fact to get the formula for the sum of the first n squares; let's call this S_n. Knowing that a square is uniquely determined by its diagonal, I drew all the diagonals I could in one direction only (Figure 2) and counted the number of squares that belonged to each diagonal. For instance, on the longest diagonal you see 1 + 2 + 3 + 4 + 5. This is because there are 5 unit squares with corners on this diagonal, 4 squares of side 2, 3 squares of side 3, and so on.

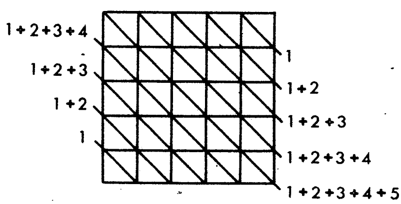

Figure 2

†From *Student Mathematics*, 1972, page 4.

Since this method counted *all* the squares and each square only *once*, I knew that S_5 was the sum of all the numbers written on the diagonals.

$$\begin{aligned} S_5 = {} & 1 + \\ & 1 + 2 + \\ & 1 + 2 + 3 + \\ & 1 + 2 + 3 + 4 + \\ & 1 + 2 + 3 + 4 + 5 + \\ & 1 + 2 + 3 + 4 + \\ & 1 + 2 + 3 + \\ & 1 + 2 + \\ & 1. \end{aligned}$$

Since this pattern looked incomplete, I completed it as in Figure 3:

$$\begin{aligned} & 1 + 2 + 3 + 4 + 5 + \\ & 1 + 2 + 3 + 4 + 5 + \\ & 1 + 2 + 3 + 4 + 5 + \\ & 1 + 2 + 3 + 4 + 5 + \\ & 1 + 2 + 3 + 4 + 5 + \\ & 1 + 2 + 3 + 4 + 5 + \\ & 1 + 2 + 3 + 4 + 5 + \\ & 1 + 2 + 3 + 4 + 5 + \\ & 1 + 2 + 3 + 4 + 5 + \\ & 1 + 2 + 3 + 4 + 5 + \\ & 1 + 2 + 3 + 4 + 5 . \end{aligned}$$

Figure 3

Now, looking at the two new triangular sections, I noticed that in each I had brought in five 5's, four 4's, three 3's, two 2's, and one 1, with a total of $5^2 + 4^2 + 3^2 + 2^2 + 1^2$, which is exactly S_5. So the sum of all the numbers in Figure 3 must be just $3S_5$. As all the rows are identical it follows that

$$3S_5 = 11 \times (1 + 2 + 3 + 4 + 5).$$

Let's turn to the general case. For an n by n square, the same procedure will give

$$3S_n = (2n + 1)(1 + 2 + 3 + \cdots + n).$$

As in $1 + 2 + 3 + \cdots + n = n(n + 1)/2$, we obtain

$$S_n = n(n + 1)(2n + 1)/6.$$

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HEADS I WIN, TAILS YOU LOSE

by G. A. Watterson, Monash University

First, you may like to check out the statement made by the writer of a letter to "The Age" on 4th June 1977.

Punter's dream

SIR, - According to Michelle Grattan's article ("The Age", 30/5) on the Labor Party leadership, the Tasmanian Liberal MHR, Michael Hodgson, would become destitute very quickly if he was a bookmaker.

His odds of 5/2 "on" for Mr Whitlam and 7/2 "against" for Mr Hayden were a punter's dream. You could put \$70 on Mr Whitlam and \$22 on Mr Hayden and still win money no matter who won the leadership contest.

If this is how Liberal politicians do their sums then how do they get on when the mathematical problem is Australia's economy?

KEN HOLMAN (Rosanna).

Now for some mathematics!

When a contest occurs between two people A and B, suppose that one of them must win (i.e. there are no ties) and that a bookmaker offers odds of $a : 1$ against A winning and $b : 1$ against B winning. [For instance, odds $7 : 2$ against can be expressed as $\frac{7}{2} : 1$ against, while odds $5 : 2$ on are equivalent to $\frac{2}{5}$: 1 against.]

If you invest an amount $\$x$ in betting that A will win and an amount $\$y$ in betting that B will win, can you be sure that you will win some money no matter whether A or B wins the contest? Let us see.

If A wins the contest, you get back your $\$x$, and you also make a net profit of $\$P$ say, where

$$P = xa - y, \tag{1}$$

that is, you get your winnings on A but lose your investment on B. Your profit, if negative, would really be a loss, of course. Similarly, if B wins, you make a profit $\$Q$, where

$$Q = yb - x. \tag{2}$$

Your possible profits will both be positive, i.e. $P > 0$ and $Q > 0$, provided that in (1), $xa - y > 0$, i.e. $\frac{x}{y} > \frac{1}{a}$, and in (2), that $yb - x > 0$, i.e. $\frac{x}{y} < b$. There is no danger in dividing by y here; y cannot be 0 and still yield a positive profit if B wins.

The bookmaker would therefore be silly if he offered odds so that you *could* choose a number $\frac{x}{y}$ greater than $\frac{1}{a}$ and less than b. He can prevent this by making

$$b \leqslant \frac{1}{a},$$

or equivalently,

$$1 - ab \geqslant 0. \tag{3}$$

The odds mentioned in the Age letter had $a = \frac{2}{5}$ and $b = \frac{7}{2}$, and then $1 - ab = 1 - \frac{7}{5} = -\frac{2}{5}$. Thus (3) is false. You could bet $\$x$ on Mr Whitlam winning and $\$y$ on Mr Hayden winning and, provided you made $\frac{x}{y}$ lie between $\frac{1}{a} = \frac{5}{2}$ and $b = \frac{7}{2}$, you would be sure of winning. Mr Holman suggested taking $\frac{x}{y} = \frac{70}{22}$ which does, indeed, lie between $\frac{5}{2}$ and $\frac{7}{2}$.

Let us now convert the problem into probabilities. If the bookmaker says the odds against A winning are a : 1, he means that A has a "probability" of winning the contest equal to $\frac{1}{a+1}$. Similarly, b : 1 against B means that B has a "probability" of $\frac{1}{b+1}$ of winning the contest. Of course, as either A or B must win, these two "probabilities" should add to 1. But in fact,

$$\frac{1}{a+1} + \frac{1}{b+1} = \frac{a+1-a}{a+1} + \frac{1}{b+1}$$
$$= 1 - \frac{a}{a+1} + \frac{1}{b+1}$$
$$= 1 + \frac{1-ab}{(a+1)(b+1)} .$$

So, if the bookmaker is not silly, he will offer you odds which correspond to "probabilities" adding up to 1 *or more*, because if (3) is true, the right hand side of (4) *is* 1 or more.

If you extend this to a contest, say a horse race, having more than two contestants, you find that if you convert a sensible bookmaker's odds into "probabilities", these "probabilities" add up to 1 or more. In fact, at the horse races, the "probabilities" add up to about 1·1 or so; check in Monday's paper! The more the "probabilities" exceed 1, the more chance the *bookmaker* has of winning, no matter which horse wins.

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MORE ON CRICKET TEAMS

In Part 1 of *Function*, on page 8, we quoted the Melbourne "Age" announcement of the winner of its Centenary Test cricket competition. The number of ways that twelve players can be chosen from 30 candidates, eleven in batting order and one as twelfth man, is

$$30 \times 29 \times 28 \times 27 \times 26 \times 25 \times 24 \times 23 \times 22 \times 21 \times 20 \times 19.$$

The number of ways that the captain and vice captain can be chosen from the eleven actually in the team is

$$11 \times 10.$$

Thus, the total number of different possible sensible entries in the competition is

$$30 \times 29 \times \cdots \times 20 \times 19 \times 11 \times 10 \simeq 4\ 557\ 300\ 000\ 000\ 000\ 000.$$

This is 10 times the figure the "Age" announced.

Any attempt to convert this calculation into a "probability of only one reader filling the requirements" is doomed to failure, of course. The possible choices of team are not equally likely (who would want eleven bowlers?), we do not know how many persons put in entries nor how many different entries were made per person. Another reader turned the tables. He wrote that the fact that only two people (Ian Johnson and the winner) selected the same team in order proved that "Both of them are pretty poor judges"!

SRINIVASA RAMANUJAN

by Liz Sonenberg, Monash University

In December 1887 a boy was born in India who became a most extraordinary mathematician. He had only a limited formal education and had, for the most part of his life, no access to books or to people who could tell him about the work that other mathematicians were doing at the time. Despite these handicaps he was able to develop an enormous amount of original mathematics. In this article we take a brief look at this extraordinary man. The information contained here comes from two obituary notices - one by G.H. Hardy, the other by Seshu Aiyar and Ramachandra Rao. These notices have been published together with the collected works of Ramanujan.

Srinivasa Ramanujan was born on the 22nd December 1887, in the Tanjore district of the Madras presidency. He first went to school at the age of five and before he was seven won a scholarship to another school. Quiet and meditative, Ramanujan was very fond of numerical calculations and had an unusual memory for numbers.

When he was 15, a friend obtained for him (from the library of the local government college) a copy of a book titled "Synopsis of Pure Mathematics". This book summarised some 6000 theorems of algebra, trigonometry, calculus and analytic geometry, generally without detailed proofs. For the most part the mathematical knowledge contained in it went no further than that of the 1860's, but it was this book that awakened Ramanujan's genius. He set himself at once to establishing its formulae. As he had no other books, each solution was for him a piece of original research. He first devised methods for constructing magic squares, then branched off to geometry and after that to algebra. According to Seshu Aiyar and Ramachandra Rao, who were two of Ramanujan's closest friends in India, Ramanujan used to say that the goddess of Namakkal inspired him with the formulae in dreams. It is a remarkable fact that, on rising from bed, he would frequently note down results and verify them, though he was not always able to supply a rigorous proof. This pattern repeated itself throughout his life.

At 16 he passed his matriculation examination to the Government College at Kumbakonam and won a scholarship. By this time he was so absorbed in his study of mathematics that he used to take no notice of what was happening in his other classes. This neglect of his other subjects resulted in failure in his examinations and he lost his scholarship. He then left Kumbakonam and went to Madras where he presented himself for another examination but failed and never tried again. Afterwards he had no very definite occupation till 1909, but continued working at mathematics in his own way. In 1909 he married and it became necessary for him to find some permanent employment. In the course of his search for work he was given a letter of recommendation to Ramachandra Rao who was then Collector at Melore, a small town 80 miles north of Madras. Ramachandra was himself a lover of mathematics and although at first he could not understand the mathematics that Ramanujan had developed on his own, he could sense that here was a man worthy of support. To allow Ramanujan time to do his mathematics, Ramachandra undertook to pay his expenses for a time. Meanwhile Ramanujan made other, unsuccessful, attempts to obtain a scholarship and being unwilling to be supported by anyone for any length of time, he accepted a minor appointment in the office of the Madras Port Trust.

But Ramanujan never slackened his work in mathematics. His earliest published works appeared in 1911 in the Journal of the Indian Mathematical Society when he was 23. By then he had made contact with other mathematicians in Madras and, on their suggestion, began corresponding with the eminent British mathematician G.H. Hardy who was then a fellow of Trinity College, Cambridge. Hardy was immediately impressed by Ramanujan's knowledge and expertise. Some of the results that Ramanujan communicated in his letters were already known to Hardy, having been discovered by other mathematicians, but many of the formulae defeated Hardy completely - he had never seen anything like them before.

In one of Ramanujan's early papers (published in 1914) there appears a very striking collection of approximations to π. For example he gives

$\frac{63}{25} \cdot \frac{17 + 15\sqrt{5}}{7 + 15\sqrt{5}} = 3{\cdot}14\ 159\ 265\ 380 \cdots$ which is correct to 9 decimal places, and $(9^2 + \frac{19^2}{22})^{\frac{1}{4}} = 3{\cdot}14\ 159\ 265\ 262 \cdots$ which is correct to 8 decimal places. Other more complicated formulae in his paper are correct to as many as 31 decimal places.

In an earlier paper Ramanujan gives a geometrical construction which yields $\frac{355}{113}$, another approximation to π. We reproduce Ramanujan's paper at the end of this article.

Ramanujan's mathematical interests were very specific. He was interested in numbers, in algebraic formulae, and in transformations of infinite series. He did not care about the possible 'usefulness' of his mathematical work in other disciplines. His intuition was most at ease in the bewildering complexities of the number system. Numbers were his friends; in the simplest array of digits he detected wonderful properties and relationships which escaped the notice of even the most gifted mathematicians. However Hardy, who probably knew more of Ramanujan than anyone else, points out that his ideas as to what constituted a mathematical proof were of the most shadowy description. All his results, new or old, right or wrong, had been arrived at by a process of mingled argument, intuition, and induction, of which he was entirely unable to give an adequately coherent account. Because of this, along with his numerous brilliant successes he had some spectacular failures - notably his work on prime numbers (to which he attached a great importance) was definitely wrong. This was explained to him only later when he was in England.

In May of 1913, because of the help of many friends, Ramanujan was able to give up his clerical post in the Madras Port Trust to take up a special scholarship. Hardy had made efforts from the first to bring Ramanujan to Cambridge. The way seemed to be open, but Ramanujan refused at first to go because of caste prejudice and lack of his mother's consent.

"This consent", wrote Hardy in his obituary of Ramanujan, "was at last got very easily in an unexpected manner. For one morning his mother announced that she had had a dream on the previous night, in which she saw her son seated in a big hall amidst a group of Europeans, and that the goddess Namagiri had commanded her not to stand in the way of her son fulfilling his life's purpose."

When Ramanujan finally came to Cambridge he had a scholarship from Madras of £250, of which £50 was allotted to the support of his family in India, and an allowance of £60 from Trinity College.

In England Ramanujan continued working on his own mathematical ideas. He conveyed to the mathematicians there the developments he had achieved and in turn he was able to learn of the work of other mathematicians - thus filling in some of the enormous gaps in his overall mathematical knowledge. In 1917 he fell ill, probably with tuberculosis, and went to a nursing home in Cambridge. After this, he was never out of bed for any length of time. However he continued working and in the last three years of his life he discovered some of his most beautiful theorems. Early in 1919 he returned home to India and died there the following year when only 33 years old.

For an evaluation of Ramanujan's work in mathematics we again quote from Hardy. "I have often been asked whether Ramanujan had any special secret; whether his methods differed in kind from those of other mathematicians; whether there was anything really abnormal in his mode of thought. I cannot answer these questions with any confidence or conviction; but I do not believe it. My belief is that all mathematicians think, at bottom, in the same kind of way, and that Ramanujan was no exception. He had, of course, an extraordinary memory. He could remember the idiosyncrasies of numbers in an almost uncanny way. It was Mr Littlewood (I believe) who remarked that 'every positive integer was one of his personal friends'. I remember once going to see him when he was lying ill at Putney. I had ridden in taxi-cab No. 1729, and remarked that the number seemed to me a rather dull one, and that I hoped it was not an unfavourable omen. 'No', he replied, 'it is a very interesting number; it is the smallest number expressible as a sum of cubes in two different ways'."

Hardy concluded his obituary notice in the following way:

"It is often said that it is much more difficult now for a mathematician to be original than it was in the great days when the foundations of modern analysis were laid; and no doubt in a measure it is true. Opinions may differ as to the importance of Ramanujan's work, the kind of standard by which is should be judged, and the influence which it is likely to have on the mathematics of the future. It has not the simplicity and inevitableness of the greatest work; it would be greater if it were less strange. One gift it has which no-one can deny - profound and invincible originality. He would probably have been a greater mathematician if he had been caught and tamed a little in his youth; he would have discovered more that was new, and that, no doubt, of greater importance. On the other hand he would have been less of a Ramanujan, and more of a European professor and the loss might have been greater than the gain."

However, in a lecture delivered at the Harvard Tercentenary Conference of Arts and Sciences in 1936, Hardy, in reviewing his earlier writings about Ramanujan, makes the following further comment:

"I have been looking again through what I wrote about Ramanujan sixteen years ago, and, although I know his work a good deal better now than I did then, and can think about him more dispassionately, I do not find a great deal which I should particularly want to alter. But there is one sentence which now seems indefensible. I wrote ..... (Hardy repeats the last paragraph of the obituary) ....., and I stand by that except for the last sentence, which is quite ridiculous sentimentalism. There was no gain at all when the College at Kumbakonam rejected the one great man they had ever possessed, and the loss was irreparable; it is the worst instance that I know of the damage that can be done by an inefficient and inelastic educational system. So little was wanted, £60 a year for five years, occasional contact with almost anyone who had real knowledge and a little imagination, for the world to have gained another of its greatest mathematicians."

∞ ∞ ∞

On the next page we reproduce a paper of Ramanujan giving a remarkable construction for (an approximation to) π. The first step in the argument, claiming that $RS^2 = (5/36)d^2$, follows from the fact that $RS = QT$ and $QT^2 = PT \cdot TR$ $(=(5/6)d \cdot (1/6)d)$. This latter equation holds because triangles QTR and PTQ are similar.

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SQUARING THE CIRCLE

by S. Ramanujan

Let PQR be a circle with centre O, of which a diameter is PR. Bisect PO at H and let T be the point of trisection of OR nearer R. Draw TQ perpendicular to PR and place the chord $RS = TQ$.

Join PS, and draw OM and TN parallel to RS. Place a chord $PK = PM$, and draw the tangent $PL = MN$. Join RL, RK and KL. Cut off $RC = RH$. Draw CD parallel to KL, meeting RL at D.

Then the square on RD will be equal to the circle PQR approximately.

For $$RS^2 = \frac{5}{36}d^2,$$

where d is the diameter of the circle.

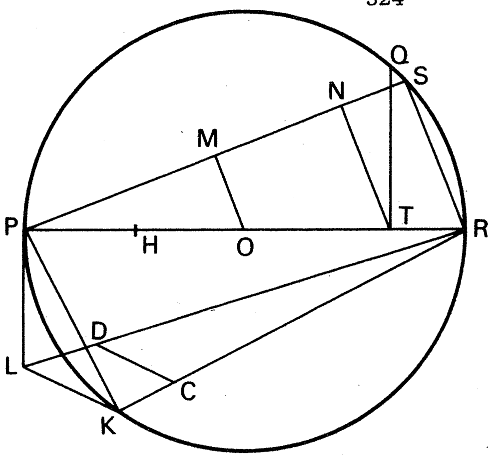
Therefore $$PS^2 = \frac{31}{36}d^2.$$

But PL and PK are equal to MN and PM respectively.

Therefore $$PK^2 = \frac{31}{144}d^2, \text{ and } PL^2 = \frac{31}{324}d^2.$$

Hence $$RK^2 = PR^2 - PK^2 = \frac{113}{144}d^2,$$

and $$RL^2 = PR^2 + PL^2 = \frac{355}{324}d^2.$$



† (*Journal of the Indian Mathematical Society*, v, 1913, p. 132)

But $$\frac{RK}{RL} = \frac{RC}{RD} = \frac{3}{2}\sqrt{\frac{113}{355}},$$
and $$RC = \frac{3}{4}d.$$

Therefore $RD = \dfrac{d}{2}\sqrt{\dfrac{355}{113}} = r\sqrt{\pi}$, very nearly.

Note. If the area of the circle be 140 000 square miles, then RD is greater than the true length by about an inch.

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MORE ABOUT π

by A.J. van der Poorten†

University of New South Wales

It is easy to check the identity
$$\frac{x^4(1-x)^4}{1+x^2} = x^6 - 4x^5 + 5x^4 - 4x^2 + 4 - \frac{4}{1+x^2}.$$
But the left side is clearly positive for all x between 0 and 1, so that the area under this curve, between $x = 0$ and $x = 1$ is positive:
$$\int_0^1 \frac{x^4(1-x)^4}{1+x^2}\,dx > 0.$$
Using the identity, this area is also
$$\int_0^1 (x^6 - 4x^5 + 5x^4 - 4x^2 + 4)\,dx - 4\int_0^1 \frac{1}{1+x^2}\,dx$$
$$= \frac{1}{7} - \frac{4}{6} + \frac{5}{5} - \frac{4}{3} + 4 - 4[\mathrm{Tan}^{-1}x]_0^1$$
$$= \frac{22}{7} - \pi.$$
As this is positive, we have proved that $\pi < \frac{22}{7}$.

[Note: The fact that $1/(1 + x^2)$ has antiderivative $\mathrm{Tan}^{-1}x$ can be proved by differentiating both sides of the identity
$$\tan(\mathrm{Tan}^{-1}x) = x.]$$

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† A.J. van der Poorten showed me the above several years ago. I hope he forgives me for communicating it to *Function*; I wanted it to appear in more permanent form than on a slip of paper in my wallet!

G.A. Watterson

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THE MAGIC HEXAGON

by M.A.B. Deakin, Monash University

Many readers will be familiar with the magic squares - arrangements like that shown in Figure 1. The nine (in this case) small squares form a larger square with the property that for each row, each column and both diagonals, the sum of the numbers involved is 15.

4	9	2
3	5	7
8	1	6

Figure 1

Another feature in this arrangement is the use of each of the consecutive numbers one to nine exactly once.

There are other magic squares. A 4 × 4 square is depicted in Durer's famous engraving *Melencolia I*.† Here the numbers 1 to 16 are arranged in such a way that each row, column or diagonal sums to 34.

The study of such magic squares can hardly be said to be a major theme of mathematics, but it is an interesting and widely known recreational topic. It becomes more and more complicated as the size of the square is increased, and much remains to be discovered, even for relatively small squares. Often amateurs surprise professional mathematicians by finding previously unknown results.

You might like to try your own hand exploring this area. For a start, calculate what the sum of the numbers should be in a 5 × 5 square, and an $n \times n$ square.

Apparently more complicated than the magic squares are the magic hexagons. Regular hexagons pack neatly as in Figure 2. Here 19 small hexagonal cells are placed together to form a shape which, while not a hexagon, has the same six-sided symmetry as a hexagon. By a slight, but allowable, misuse of language,

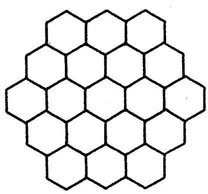


Figure 2

† Made in 1514. On display at the British Museum.

this figure is referred to as a hexagon: in this case a hexagon of order three, as there are three cells on each side.

If we examine the structure of the hexagon in Figure 2, we see that there are 5 horizontal rows of cells, 5 rows slanting from top left to bottom right of the page and 5 slanting from top right to bottom left. There are 19 cells in all arranged in a total of 15 rows.

The problem is to arrange the numbers 1 to 19 in the cells of Figure 2 so that the sum along each row is the same as the sum along every other row. We're not asking you to do this (for reasons which will become obvious), but you could try to see why each row must add up to 38.

The answer to the arrangement problem is usually attributed to Clifford Adams, an amateur mathematician who may be said, without exaggeration, to have devoted half a lifetime to its solution.

Adams, a railway clerk, began his search in 1910. He had a set of hexagonal ceramic tiles specially made, each bearing a number from 1 to 19, and used these in an experimental effort of mammoth proportions. (Disregarding the different points of view achieved by rotations and reflections, how many combinations are there?)

His spare time was devoted to this problem for 47 years. He finally found a solution while convalescing following an operation and jotted it down on a piece of paper. When he returned home, however, he found that he had mislaid the solution.

It attests to his determination that for five years, he continued (he had by then retired) his efforts to reconstruct the solution. He never succeeded. Instead, he had the good luck to locate the missing piece of paper.

He forwarded a copy to Martin Gardner, the Scientific American columnist, in December 1962. (If you don't know Gardner's columns and the Problem Books he compiles from them, you have a treat in store.)

Let Gardner now take up the story:

"When I received this hexagon from Adams, I was only mildly impressed. I assumed that there was probably an extensive literature on magic hexagons and that Adams had simply discovered one of the hundreds of order-3 patterns. To my surprise a search of the literature disclosed not a single magic hexagon. I knew that there were 880 different varieties of magic squares of order 4, and that order-5 magic squares ... [had not then] ... been enumerated because their number runs into millions. It seemed strange that nothing on magic hexagons should have been published."

Gardner contacted Charles W. Trigg, a United States mathematician with a wide reputation in the area of combinatorics (the branch of mathematics involved) and asked for his opinion. Trigg took a month to reply, but the answer was worth waiting for.

Apart from trivial alterations caused by rotation or reflection, *Adams' magic hexagon was the only one that could exist.*

Well, not quite. There is one other. Here it is: 

This is so trivial that we don't count it. It is easy to see also that there is no magic hexagon of order 2. Suppose we have an order-2 hexagon as shown in Figure 3. The numbers one to seven must be arranged in the cells so that nine different sets of numbers all add up to the same figure. Suppose the top entries are a and b, as shown. Then all rows must add up to $a + b$, whatever that may be. But now what are we to put in the far left cell? We have:

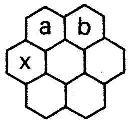


Figure 3

$$a + x = a + b$$

so that

$$x = b,$$

and the number b is used twice, which is against the rules.

(An alternative impossibility proof notices that the row sum must be 28/3. Can you produce this proof?)

Two more things remain to be proved in order to show that Trigg's theorem (if we may so term the uniqueness claim) is true. We need to be assured that:

(1) There is no magic hexagon of order n, if $n > 3$.

(2) Among all the (how many did you get?) possibilities of order 3, only one is magic.

At first sight, we would think that the first statement, which comprises infinitely many cases, would be harder to prove than the second. In point of fact, this is not the difficult part. The proof is a little long to include in this article, and contains some ideas that will be new to, but not above the capabilities of, the readers of *Function*. Interested readers will find it on pages 71-73 of Ross Honsberger's *Mathematical Gems*. (A more cryptic account is given by Martin Gardner, *Scientific American*, August 1963, p. 116.)

It remained to Trigg to show that of all the (?) possibilities of order 3, only one was magic. This he accomplished in a proof that, on Gardner's account, "..... used a ream and a half [750 pages] of sheets on which the cell pattern had been reproduced six times", i.e. the "answer was obtained by combining brute force ..... with clever short cuts".

That short cuts were necessary may be seen easily enough. There are (?) possible combinations, of which Trigg needed to discuss only 6 × 750.

The case is somewhat reminiscent of that discussed by John Stillwell in the first issue of *Function*. In discussing the four colour problem, he referred to the Haken-Appel solution as "a barbarous way to do mathematics", and our editorial indicated that some check was necessary before the result could be unprovisionally accepted.

Trigg's theorem provides a similar case. The result is not important enough for anyone to pay for its publication. It is no slight on Gardner to say that he probably did not check all the details. Are we then to hold Trigg's theorem unproved, or only probably right?

In this case, the answer is "no". The result was proved independently by Frank Allaire (in 1969). Allaire was then a second-year student at the University of Toronto. Using an elegant computer programme, Allaire reduced the problem to 70 cases (each involving many sub-cases) and confirmed Trigg's theorem in 17 seconds of computer time. Enough of his method is now public (see, e.g., pp. 73-76 of *Mathematical Gems*) that any bright young mathematician with a flair for combinatorics and computing can check the result.

Trigg and Allaire thus not only duplicated the result of Adams' search, but extended it. Trigg (without a computer by the way) did more in a month than Adams achieved in 47 years. However, just as Allaire knew from Trigg's work what he had to aim for, so Trigg knew from Adams' more pioneering efforts where he was going. (Trigg's "clever short cuts", the result of a well-practised mathematical mind, had much to do with this also.) Did Adams himself have some guiding star? It appears now that he may have done. Gardner more recently (in *University of Chicago Magazine*, Spring, 1975) shows a picture of a puzzle incorporating the magic hexagon. This was patented in 1896 by William Radcliffe, a schoolteacher on the Isle of Man. Was Adams influenced by a (possibly subconscious) memory of Radcliffe's puzzle?

Two other possible discoverers exist, although they too may owe a debt to Radcliffe. An unpublished manuscript from wartime Germany (1940) contains the result. The author is Martin Kühl of Hanover.

More ironically, a lot of the time Adams was agonising over his lost paper, the result was in print, widely distributed, but unrecognized. It is published, as a diagram, with no words at all, in *Mathematical Gazette* (1958), p. 291. The author of the strangely silent article was Tom Vickers. Perhaps the reason that Vickers' result was overlooked is the one which will strike any reader who cares to look it up. Visually, it's quite different from Figure 2. (For a start, it's a *hexagon* and not just a courtesy hexagon.)

Another reason is more subtle, but perhaps more important. Vickers' paper was overlooked, just as Adams' paper was almost dismissed by Gardner. Both were seen at first as quaint numerical curiosities. These are not, as such, of great mathematical interest. They merit attention, however, when we can say something truly surprising about them - when we "embed" them in deeper, more general, results. This is what Trigg did in this instance. Once he had done that, people took notice.

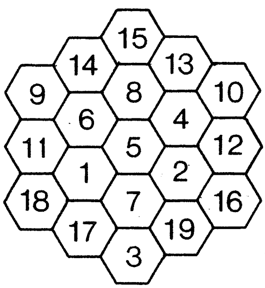
At the beginning of this article, I referred to magic hexagons as being "apparently more complicated than magic squares". They are not really more complicated, however. There is only one of them. Meanwhile, results on magic squares proliferate and we have no comprehensive framework in which to place them. Provision of such frameworks is what mathematics is about.

Here are two problems to consider:

(a) Equilateral triangles can fit together to form larger equilateral triangles; small squares combine into larger squares; regular hexagons interlock to make up "hexagons". No other regular plane figures can do this. Why not?

(b) We have talked of magic hexagons and magic squares. What of magic triangles?

P.S. You didn't really think I was condemning you to 47 years' hard labour, did you? Here is the magic hexagon:



算石子

TZIAN – SHI – ZI

Tzian-Shi-Zi[†] is a Chinese game. Two piles of objects, for instance stones are prepared. The game is played by two persons. Each of them has to take alternately at least one stone. At each turn a player may take any number of stones from one pile, but if he wishes to take stones from both piles he has to take the same number of stones from each pile. The player who takes the last stone wins.

Let us write (a, b) to denote that there are a stones in one pile and b stones in the other. We shall assume that $a \leqslant b$. It is obvious that in the cases (0, 1) and (n, n) the player who makes the next move can win. In the case (1, 2) the player whose turn it is to play loses the game. This can be verified by considering all possible cases. Let us call the pair (1, 2) an *S–L* pair (starter-loses). On the other hand in the case $(1, n)$ with $n \geqslant 3$, the starting player may win by taking $n - 2$ stones from the second pile and leaving the combination (1, 2) for his opponent. The question is how to find all possible *S–L* combinations. If a player knows them, then with each of his moves he should try to leave for his opponent an *S–L* combination and thus win the game.

It is obvious that no pair $(2, b)$, $b \geqslant 2$ is an *S–L* combination. Indeed here the starting player takes $b - 1$ stones and wins the game. On the other hand the pair (3, 5) is an *S–L* combination. The starting player A cannot leave his opponent the combination (2, 1), so if he takes at least one stone from the first pile, his opponent B will win the game.

In the remaining cases player B should move as follows

A	(3, 4)	(3, 3)	(3, 2)	(3, 1)	(3, 0)
B	(1, 2)	(0, 0)	(1, 2)	(2, 1)	(0, 0)

where in the top row the combination left after A's move is shown and in the bottom row B's move is described in the same way.

PROBLEM (Solutions invited please)

Can you find a rule to obtain all possible *S–L* pairs? (There are infinitely many *S–L* pairs.) Try, then, to prove that if a player A faces a pair which not an *S–L* pair then he can make a move in such a way that his opponent will face an *S–L* pair.

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† Count stones small

A NOTE ON CALENDARS

by Mark Michell, Lynwood Avenue, East Ringwood

In the first issue of *Function*, in an article called 'A Perpetual Calendar', a formula was given which enables you to work out on which day of the week a given date falls. Here we use the formula to work out which days of the week can occur as the first day of a century.

The formula we use comes from the bottom of page 22 in the first issue of *Function*. We are interested in day 1 of month 1 (i.e. January) of year $J \times 100$. The given formula says that for this date

$$x \equiv 1 + \left[\frac{26(13+1)}{10}\right] + 99 + \left[\frac{99}{4}\right] + \left[\frac{J-1}{4}\right] - 2(J-1) - 1$$

where x is the day of the week on which the given date falls, so that Sunday is day 0, Monday is day 1 etc. (The meaning of $\equiv$ and [ ] is explained in 'A Perpetual Calendar': $a \equiv b$ means that $a - b$ is divisible by 7; $[c]$ denotes the integral part of c.)

After some calculation we get

$$x \equiv 161 + \left[\frac{J-1}{4}\right] - 2J.$$

Using the rule for calculating with residues which was worked out in 'A Perpetual Calendar', we have, since 161 is a multiple of 7,

$$x \equiv \left[\frac{J-1}{4}\right] - 2J\,. \qquad (1)$$

Let us look at a couple of particular dates and work out x. For example 1st January 2000 and 1st January 2400.

If $J = 20$ then from formula (1) we have $x \equiv \left[\frac{19}{4}\right] - 40 = 4 - 40 = -36$. Using the rule worked out in 'A Perpetual Calendar' (extended so as to apply to negative numbers also) we obtain $-36 \equiv 6$, since the difference $6 - (-36) = 42$ is a multiple of 7. Thus $x \equiv 6$, i.e. January 1st 2000 will be a Saturday.

If $J = 24$ then from (1) we have $x \equiv \left[\frac{23}{4}\right] - 48 = -43$. As $6 - (-43) = 49$ is a multiple of 7 we obtain $-43 \equiv 6$ and so $x \equiv 6$, i.e. January 1st 2400 will be a Saturday also.

Similar calculations give the following table

Century	First day of Century
21st (i.e. 2000)	Saturday
22nd	Friday
23rd	Wednesday
24th	Monday
25th	Saturday
26th	Friday
27th	Wednesday
28th	Monday

This suggests that Sunday, Tuesday and Thursday never occur as the first day of a century. Can we *prove* it?

Let $F(J) = [\frac{J-1}{4}] - 2J$. Then

$$F(J+4) = [\frac{(J+4)-1}{4}] - 2(J+4)$$

$$= [\frac{J-1}{4} + 1] - 2J - 8$$

$$= [\frac{J-1}{4}] + 1 - 2J - 8$$

$$= [\frac{J-1}{4}] - 2J - 7$$

Thus $F(J+4) - F(J) = -7$ so that $F(J) \equiv F(J+4)$. This says that every four centuries the same day occurs at the beginning of the century. Thus the four days worked out above (Monday, Wednesday, Friday, and Saturday) are the only days that will occur as the first day of the century.

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SOLUTION TO PROBLEM 1.1.

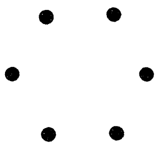
To solve this problem we have to find how often the thirteenth falls on each day of the week. From the article 'A Perpetual Calendar' by Dr E.A. Sonenberg, in Part 1 of *Function* we know that the whole calendar repeats itself every 400 years. Using the formulae given in that article we calculate (the calculation is a little long) that, in any 400-year period, for example from January 1st 1600 to December 31st 1999, inclusive, the 13th of the month falls on each day with the frequencies given in the following table:

Sun	Mon	Tues	Wed	Thur	Fri	Sat
687	685	685	687	684	688	684 .

So the 13th does indeed fall on a Friday more frequently than it falls on any other single day of the week.

THE GAME OF SIM

The game of SIM is played by two players each with a coloured pencil. Start with six points drawn on a piece of paper as follows



Each player in turn draws a line joining a pair of the points. The first player to form a complete triangle of his own colour, *loses*. (Only triangles with all vertices on the six starting points are considered and no pair of points may be used more than once.) The object of the game is not to form such a triangle yourself and to force your opponent to do so.

Since at each turn one pair of points is 'used up' and there are only $\binom{6}{2} = \frac{6 \times 5}{2 \times 1} = 15$ different pairs of points, each game is quite short. Further, unlike the game of noughts and crosses for example, it seems quite difficult to find good strategies. Also as we explain below it can be shown that a drawn game is impossible. These two things make the game of SIM quite a good time-waster.

We show that a drawn game is impossible by showing that if all fifteen lines are drawn in with two colours (say red and blue) then there must be a triangle in one of the colours. For consider any one point, P say, of the six points after all 15 lines have been drawn in. Since five lines originate at P (one going to each of the other given points), at least three of these lines must be of the same colour. Notice that we can't say *which* of the two colours it is, only that these lines have the *same* colour. Let us suppose that it is the colour blue. (We could give a similar argument starting with the colour red.) Let us give names to three of the lines from P which have the colour blue - PQ, PR, and PS say. If QR is blue then PQR is a blue triangle. Similarly, if either of RS or SQ is blue, we can find a blue triangle. But if none of QR, RS, or SQ is blue then they must all be red, i.e. QRS is a red triangle. Thus as required there is always a triangle in one of the two colours.

"Ah! why, ye Gods, should two and two make four?"

Alexander Pope: *The Dunciad*

SOLUTION TO PROBLEM 2.4.

"The one on the left can't be Tom because Tom always tells the truth and if he was Tom he would be saying *I'm Tom* and he is saying *The guy in the middle is Tom*. The one in the middle can't be Tom either because he is saying *I'm Dick* and Tom always tells the truth. So the one on the right must be Tom.

Tom is saying *the guy in the middle is Harry* so he must be because Tom always tells the truth so the one on the left must be Dick!"

Solution by Gay Deakin, Grade 4, Blackburn Lake State School. Also solved, by a different argument, by Julie Deakin, Grade 6, of the same school.

SOLUTION TO PROBLEM 2.5.

For the general case:

$$a_n a_{n-1} \cdots a_3 a_2 a_1 a_0 \times h/k = a_0 a_n a_{n-1} \cdots a_3 a_2 a_1.$$

Then $a_n(h10^n - k10^{n-1}) + a_{n-1}(h10^{n-1} - k10^{n-2}) + \cdots$

$\cdots + a_2(h10^2 - k10) + a_1(h10 - k) + a_0(h - 10^n \times k) = 0.$

Since $h10^i - k10^{i-1} = h \times 10 \times 10^{i-1} - k \times 10^{i-1} = (10h - k)10^{i-1}$

we have $a_0(k10^n - h) = (10h - k)(a_n 10^{n-1} + a_{n-1}10^{n-2} + \cdots$

$$\cdots + a_3 10^2 + a_2 10 + a_1) = (10h - k)x, \text{ say.} \qquad (1)$$

i) $\frac{h}{k} = \frac{3}{2}$

Equation (1) here gives $a_0(2 \times 10^n - 3) = 28x$. However $2 \times 10^n - 3$ is never divisible by 28, but when $n = 5 + 6p$, for $p \geqslant 0$, an integer, $2 \times 10^n - 3$ is divisible by $7 = \frac{28}{4}$ so if we let $a_0 = 4$, then $4(2 \times 10^n - 3) = 28x$, i.e. $2 \times 10^n - 3 = 7x$.

For $n = 5$, $x = 28\ 571$, for $n = 11$, $x = 28\ 571\ 428\ 571$. Since $a_0 = 4$, solutions to the problem are

$$285\ 714 \times \frac{3}{2} = 428\ 571$$

and

$$285\ 714\ 285\ 714 \times \frac{3}{2} = 428\ 571\ 428\ 571.$$

Also $a_0 = 8$, provides an additional set of solutions

$$571\ 428 \times \frac{3}{2} = 857\ 142 \text{ etc.}$$

ii) $\frac{h}{k} = \frac{7}{4}$

Equation (1) now gives

$$a_0(4 \times 10^n - 7) = 66x. \qquad (2)$$

If we divide $4 \times 10^n - 7$ by 66 we find that we get a remainder 63 when n is even (note $-3 = (-1) \times 66 + 63$) and a remainder 33 when n is odd. Hence equation (2) has solutions if a_0 is even and n is odd. Taking $a_0 = 2, 4, 6, 8$ in turn, with $n = 1$ we therefore get solutions

12, 24, 36, 48;

with $n = 3$, we get

1212, 2424, 3636, 4848;

and so on.

The above solution is by Mr G.J. Chappell of "Bethany", Rubyanna Road, Bundaberg, Queensland. Mr Chappell politely does not point out that we made a mistake in (i) where we stated that no solution was possible.

SOLUTION TO PROBLEM 1.2. (Fibonacci's problem.)

The answer (found by Leonardo Fibonacci) is $\frac{41}{12}$. It may be checked that

$$(\tfrac{41}{12})^2 + 5 = (\tfrac{49}{12})^2$$

and

$$(\tfrac{41}{12})^2 - 5 = (\tfrac{31}{12})^2.$$

SOLUTION TO PROBLEM 1.6.

No two statements can be simultaneously correct, for the mth statement requires that precisely m of the statements are incorrect. Hence at least 99 of the statements must be incorrect. If all 100 statements were incorrect then the 100th statement would be correct, a contradiction. Hence precisely 99 statements are incorrect. Thus the 99th statement is the only correct statement.

SOLUTION TO PROBLEM 1.5.

Yes, the statement that the New Zealand dollar has been revalued by 12·7% in comparison with the Australian dollar, is correct.

Suppose that, before the devaluations, one New Zealand dollar costs k Australian dollars. When Australia devalues by 17½% each Australian dollar is then worth 82½% of what it was worth before devaluation. It thus takes $\frac{100}{82\frac{1}{2}}$ of the new dollars to buy what one of the old dollars bought. Hence in devalued Australian dollars, one New Zealand dollar now costs $(k \times \frac{100}{82\frac{1}{2}})$ dollars.

When New Zealand devalues by 7% it only costs $\frac{93}{100}$ of the previous cost to buy a devalued New Zealand dollar. Hence the cost of the devalued New Zealand dollar is $\frac{93}{100} \times \frac{100}{82\frac{1}{2}} \times k$ devalued dollars.

Since $$\frac{93}{100} \times \frac{100}{82\frac{1}{2}} = \frac{112\cdot\dot{7}\dot{2}}{100},$$

the number of Australian dollars now required to buy a New Zealand dollar is $12.\dot{7}\dot{2}\%$, i.e. approximately $12\cdot7\%$ more than before the devaluations took place.

Notice that the result of the calculation is unaffected by the order in which the devaluations take place. If New Zealand's devaluation took place first and was then followed by Australia's then the new cost of one New Zealand dollar would be $\frac{100}{82\frac{1}{2}} \times \frac{93}{100} \times k$ Australian dollars.

PROBLEM 3.2.

How could a car make the skid marks as indicated on the sign?

PROBLEM 3.3.

Show that, for all integers a, b, $ab(a^2 - b^2)(a^2 + b^2)$ has 30 as a factor.

Submitted by Rob Saunders, Rusden State College

PROBLEM 3.4.

A large textbook has every page numbered. The printer used 1890 digits to number the pages. How many pages were there?

PROBLEM 3.5.

A die is thrown until a 6 is obtained. What is the probability that 5 was *not* thrown, meanwhile?

PROBLEM 3.6.

If a record is played at $33\frac{1}{3}$ r.p.m., and three musical notes are heard, namely middle C, E and G, what will the three notes be if

(i) the same record is played at 45 r.p.m.,

(ii) it is played at 78 r.p.m.?

Submitted by Andrew Fortune, Arts II, Monash

PROBLEM 3.7.

Of three prisoners, Mark, Luke and John, two are to be executed, but Mark does not know which. He therefore asks the jailer 'Since either Luke or John are certainly going to be executed, you will give me no information about my own chances if you give me the name of one man, either Luke or John, who is going to be executed.' Accepting this argument, the jailer truthfully replied 'Luke will be executed'. Thereupon, Mark felt happier because before the jailer replied his own chances of execution were 2/3, but afterwards there are only two people, himself and John, who could be the one not to be executed, and so his chance of execution is only ½.

Is Mark right to feel happier?

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A FACT

Think of a word, for example, Llanfairpwllgwyngyllgogerychwyrndrobwllllantysiliogogogoch, the name of the well-known Welsh town. Count the number of letters in the word, in our case fifty eight. Now count the number of letters in the words fifty eight. We get ten. Now count the number of letters in the word ten. We get three. Count the number of letters in three. Answer five. Count the number of letters in five. Answer four. Count the number of letters in four. Answer four. And so on.

This process, whatever word we start with always terminates with fours.

Submitted by Cynthia Kelly, Science III, Monash

PROGRAM FOR FRONT COVER

by Michael Eliott, 4th Form, Scotch College

```
0001       DIMENSION K(100),L(20),X(40)
0002       DATA L(1),L(2),L(3) /1H.,1H/,1HI/
0003       DATA L(4),L(5),L(6) /1H+,1H3,1H- /,Y/36./
0004       DATA L(7),L(8),L(9),J1 /1HW,1H,,1H*,8/
0005       J2=20+J1
0006       X(J2+1)=101.
0007       PRINT 6
0008     6 FORMAT(1H1)
0009       DO 7 N=1,73
0010     9 Z=SIN(Y/10.)*10.
0011       DO 1 I=1,J1
0012       V=I
0013     1 X(I)=50.-V*Z
0014       DO 2 I=21,J2
0015       W=I-20
0016     2 X(I)=50.+W*Z
0017       J=J1
0018       M=J1
0019       DO 4 I=1,50
0020       A=I
0021     5 IF(A.LE.X(J))GO TO 4
0023       IF(J.LE.1) J=22
0025       J=J-1
0026    33 M=M-1
0027       GO TO 5
0028     4 K(I)=L(M+1)
0029       M=0
0030       J=21
0031       DO 11 I=51,100
0032       B=I
0033    22 IF(B.LT.X(J))GO TO 11
0035       J=J+1
0036       M=M+1
0037       GO TO 22
0038    11 K(I)=L(M+1)
0039       PRINT 3,K
0040     3 FORMAT(1X,120(A1))
0041       IF(N.LT.37)GO TO 7
0043       Y=Y+2.
0044     7 Y=Y-1.
0045       PRINT 6
0046       STOP
0047       END

TOTAL ERRORS  =  0000

TIME =     16
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Angling may be said to be so like the mathematics, that it can never be fully learnt.

Izaak Walton: *The Compleat Angler*, 1655

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I have yet to see any problem, however complicated which, when looked at the right way, did not become still more complicated.

Paul Anderson

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In the beginning of algebra, even the most intelligent child finds, as a rule, very great difficulty. The use of letters is a mystery, which seems to have no purpose except mystification. It is almost impossible, at first, not to think that every letter stands for some particular number, if only the teacher would reveal *what* number it stands for. The fact is, that in algebra the mind is first taught to consider general truths, truths which are not asserted to hold only of this or that particular thing, but of any one of a whole group of things. It is in the power of understanding and discovering such truths that the mastery of the intellect over the whole world of things actual and possible resides; and ability to deal with the general as such is one of the gifts that a mathematical education should bestow.

Bertrand Russell: *The Study of Mathematics*, 1902

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MATHEMATICS LECTURES

The series of lectures for 5th and 6th form students at Monash University continues. They are on Friday evenings, from 7 p.m. to 8 p.m. Your school has received a detailed programme.

Lectures are held in the Rotunda Lecture Theatre R1 (enquire at the main gate). The remaining lectures are:

July 8	Stonehenge and Ancient Egypt; the mathematics of radiocarbon.	Dr R.M. Clark
July 22	Computing Orbits.	Dr J.O. Murphy
August 5	How Things Begin; the development of some mathematical concepts.	Professor J.N. Crossley

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