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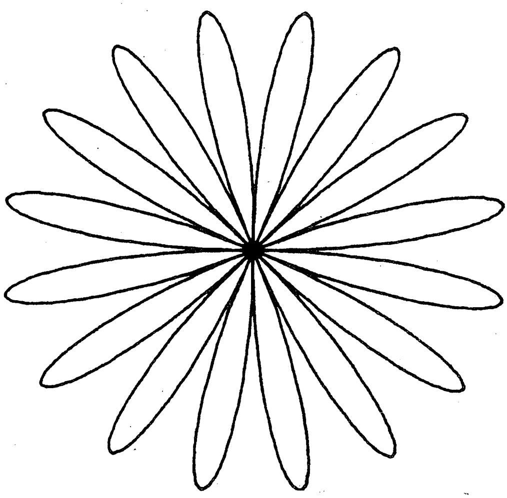

A SCHOOL MATHEMATICS MAGAZINE

Published by Monash University

Function is a mathematics magazine addressed principally to students in the upper forms of schools. Today mathematics is used in most of the sciences, physical, biological and social, in business management, in engineering. There are few human endeavours, from weather prediction to siting traffic lights, that do not involve mathematics. *Function* will contain articles describing some of these uses of mathematics. It will also have articles, for entertainment and instruction, about mathematics and its history. There will be a problem section with solutions invited.

It is hoped that the student readers of *Function* will contribute material for publication. Articles, ideas, cartoons, comments, criticisms, advice are earnestly sought. Please send to the editors your views about what can be done to make *Function* more interesting for you.

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Articles, correspondence, problems (with or without solutions) and other material for publication are invited. Address them to:

The Editors,
Function,
Department of Mathematics,
Monash University,
Clayton, Victoria, 3168

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Front cover: $r = a \sin 8\theta$

(see page 2)

The first issue of *Function* has received a warm welcome and we thank the large number of correspondents who sent their good wishes and encouragement. However we have not yet received enough subscriptions to guarantee that we last out the year. The magazine is not receiving financial help from Monash University or any other source. If you wish to see us continue - one subscription from each school library would almost suffice - please canvass subscriptions for us.

Our main article this issue is on catastrophe theory, perhaps misleadingly named. The French word "catastrophe" used by the French originator of this theory has no adequate English equivalent. A (French) catastrophe occurs when there is a sudden and perhaps unpredictable or uncontrollable change, but such a change need not be any kind of (English) catastrophe. Catastrophe theory is one of the exciting new developments of mathematics that has taken place largely in the last ten years. It developed originally as an attempt to develop mathematics capable of describing biological growth. Some of its many other areas of possible application are mentioned in Dr Deakin's article.

CONTENTS

THE FRONT COVER

by J. O. Murphy, Monash University

The so-called rose curves (rhodoneae) are generated by the polar equations

$$r = a \cos n\theta \text{ or } r = a \sin n\theta, \quad 0 \leqslant r \leqslant a, \quad 0 \leqslant \theta \leqslant 2\pi;$$

for each n these two curves are congruent and each can be obtained from the other by a rotation through $\frac{\pi}{2n}$ radians. The computer drawn diagram on the front cover illustrates $r = a \sin 8\theta$, which has 16 leaves. In general $r = a \sin n\theta$ has $2n$ leaves if n is even, and only n leaves if n is odd. The one petal case, $r = a \sin \theta$, which has the corresponding equation

$$x^2 + (y - \frac{a}{2})^2 = \frac{a^2}{4},$$

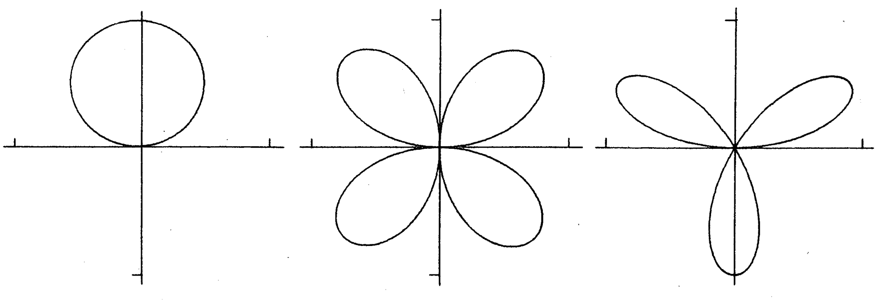
in rectangular coordinates, clearly generates a circle of diameter a.

A simple integration, using the general formula

$$A = \frac{1}{2}\int_{\theta=\alpha}^{\theta=\beta} r^2 d\theta,$$

for area in polar coordinates, establishes the area of each petal, and in the particular case, $r = a \sin 8\theta$, it is found to be $\frac{a^2\pi}{32}$ square units. Now a general problem, which illustrates a geometric property of the rose curves: show that, for each even integer n the total area enclosed by all the petals of $r = a \sin n\theta$ is always the same, namely, $\frac{1}{2}\pi a^2$. If n is odd, the area enclosed by all the petals is, similarly, always the same, but now equals $\frac{1}{4}\pi a^2$.

The rose curves $r = a \sin n\theta$, for $n = 1$, 2 and 3 are shown below.



CATASTROPHE THEORY

by M. A. B. Deakin, Monash University

According to its inventor, the French topologist René Thom, Catastrophe Theory is more an attitude of mind than a mathematical theory. Even if he is right in this, however, there is a lot of valid and interesting mathematics in the area. Not only is the mathematics new and exciting in its own right, but it has also attracted a lot of attention because of its apparent potential for applications.

Underlying this potential is a postulate that constitutes Thom's "attitude of mind". We may express this epigrammatically as *Nature is almost always well-behaved.* In a very general way, we can represent a natural process as in Figure 1. The output, or result, generally depends in a smooth and unremarkable way on the input, or conditions applied.

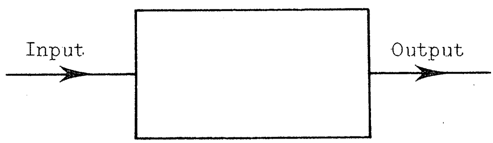


Figure 1

For instance, the density of a liquid depends in a smooth and relatively simple way on its temperature. The precise formula expressing the dependence may be very complicated, not expressible in familiar form, or even unknown, but the curve is an ordinary graph, such as Figure 2 which shows the behaviour of water.

Smooth functions such as this are studied by the use of calculus. For our present purposes, we may say that the essential insight provided by calculus is the result: *sufficiently close to any point P on its graph, a function may be approximated to arbitrarily good accuracy by a straight line, namely the tangent at P.*

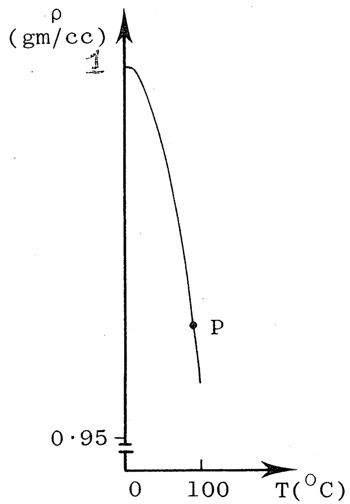


Figure 2

In our example, ρ is the density, T the temperature, and ρ_0, T_0 are the values of these variables at P. The line then has the formula

$$\rho - \rho_0 = -\beta\rho_0(T - T_0),$$

where β is a constant known as the coefficient of bulk expansion. Formulae such as this are everyday fare and depend, as this one does, on the tangent (or local linear) approximation.

In the neighbourhood of certain special points *(stationary points)* the local linear approximation is constant - i.e. the tangent is flat. (The curve of Figure 2 has a stationary point at $T = 4$.) However, the local linear approximation tells us little about the shape of the curve. For further information we must look at a local quadratic approximation. If the curve can be approximated locally by a parabola whose shape is like that of $y = x^2$, we have a minimum point; if the shape resembles that of $y = -x^2$, we have a maximum point. (We can also use a similar analysis at points other than singular points to achieve a better approximation than the local linear one.)

As a matter of fact, all but a few rather unusual maxima and minima may be locally approximated by parabolae. (The n-dimensional generalisation of this statement is known as Morse's Lemma; it is the starting point of a branch of mathematics known as Morse Theory which is closely related to Catastrophe Theory. Morse Theory is not, however, related in any way to Morse Code. It is named after its inventor, the American mathematician Marston Morse.)

A related result (which you should be able to prove) states that every quadratic $y = ax^2 + bx + c$ may, by suitable choice of origin and by suitable scaling of the axes, be reduced to the form $y = x^2$. (So all parabolae can be drawn using a single template!) This theorem, incidentally, was in the Victorian school curriculum 25 years ago.

When we come to cubics, something more interesting happens. Any cubic equation $y = Ax^3 + Bx^2 + Cx + D$ can be reduced to the form $y = x^3 + ax$ by suitable placement of the origin and suitable scaling of the axes. (Can you prove this?)

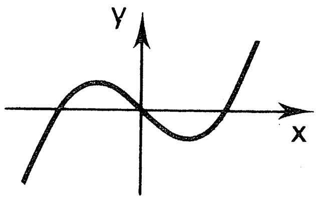


Figure 3

However, the shape of the graph now depends upon the sign of a. If $a < 0$, then the graph is as shown in Figure 3, while if $a > 0$, the appearance is that of Figure 4.

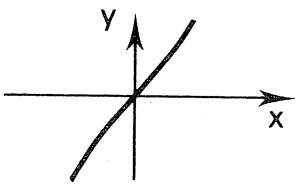


Figure 4

We may now imagine a situation such as that shown in Figure 5. Here a particle is able to rest at the minimum point P as long as that minimum exists; that is, as long as $a < 0$.

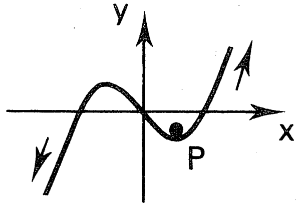


Figure 5

But now suppose that the value of a is progressively increased, e.g. by slowly pulling on the curve in the direction of the arrows. Eventually, we reach a configuration for which $a = 0$ and the minimum ceases to exist. The particle now falls to the left. A quite continuous input has produced a sudden jump in the output. The particle will fall until it encounters another minimum (beyond the range of validity of the local cubic approximation).

Such a sudden change is termed a *catastrophe*. Catastrophes are the exceptions to the orderly behaviour of nature.

The density of water manifests a similar behaviour, for the water boils if heated sufficiently. Figure 6 shows a more complete graph than Figure 2. (Note that the scale has had to be altered. The density of steam is about 6×10^{-5} gm/c.c., and this is so small as to be indistinguishable from zero on either scale. The slight lean of the apparently vertical line is barely visible.)

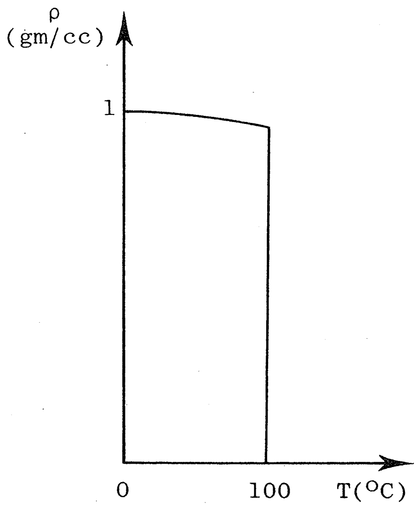


Figure 6

The boiling point is immediately obvious on this diagram because of the abnormal appearance of the graph there. This is an example of a catastrophe also.

The systematic study of such exceptions to the orderly behaviour of nature forms the subject matter of the new science of Catastrophe Theory. It is a very recent area of mathematics, which first burst into prominence with the publication of Thom's book *Structural Stability and Morphogenesis* in 1972.

The basic theorem of Catastrophe Theory gives a classification of the various types of catastrophe that can occur in certain important (but still far from completely general) situations. Its basic assumption is that the state of the system under study can be determined by maximising or minimising some suitable function. In statics, for example, the configuration achieved *minimises potential energy*; in dynamics, a quantity termed *action* *is minimised*; in thermodynamics, *entropy is maximised*; in geometrical optics, the *total travel time of a light ray is minimised*; in many areas of genetics, *mean population fitness is maximised*, and in managerial science, many firms seek to *maximise profit*. (It is the thermodynamic case that covers our example of boiling water.)

A catastrophe occurs in such a context when the system adjusts from one maximum or minimum to another, or when a previously available maximum or minimum suddenly ceases to exist.

The very simplest catastrophe is the one we saw earlier with the stretching cubic. The position of the minimum point P can be found by a straightforward differentiation. Its x-coordinate, x_P, is given by

$$x_P = \sqrt{\frac{-a}{3}} .$$

This gives the graph drawn in Figure 7. (The dotted line corresponds to the maximum in Figure 3.) The shape of this graph has led to catastrophes of this type being termed *fold catastrophes*, also referred to sometimes as *threshold catastrophes*.

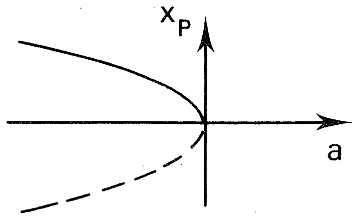


Figure 7

In this example, the value of x_P defines for us the state of the system. It is referred to as a *state variable*. The value of x_P depends upon the value of a. This latter is under our control, and is termed a *control variable*. The control variable (or more generally, variables) correspond to the input of Figure 1, while the state variable (or variables) may be thought of as the output.

All situations that involve a single control variable can, as a consequence of Thom's classification theorem, be approximated locally by a cubic. The fact that the approximation is *local* stops the particle P in Figure 5 from necessarily falling forever; it is allowed that another minimum exist in what my 5-year old son would call "the far distance". With this proviso, it is possible to set up an exact analogy between the boiling water and a variant of the stretching cubic.

However, we shall follow a different road here, for we are still oversimplifying the behaviour of the water. The density of a fluid depends not only on the temperature, but also on the pressure. Hence a full analysis uses two control variables (temperature and pressure) and one state variable (density).

According to Thom's theorem, this new situation can always be approximated locally by a quartic function

$$y = x^4 + ax^2 + bx, \qquad (*)$$

where the minima of y will give values of a state variable x_P and a, b are the control variables. (Can you show that all quartics can be reduced to the above form by suitable choice of scales and origin?)

Putting the derivative $\frac{dy}{dx}$ equal to zero, we find the stationary values of the quartic function (*)[†]. Denote by x_P the corresponding value(s) of x. Then

$$4x_P^3 + 2ax_P + b = 0.$$

This equation is a cubic in x_P. For some values of a, b, it will have three solutions (2 minima and a maximum), for others there will be only a single solution (which is a minimum). The graph of x_P as it depends upon a, b is the complicated-looking pleated surface shown in Figure 8. It is

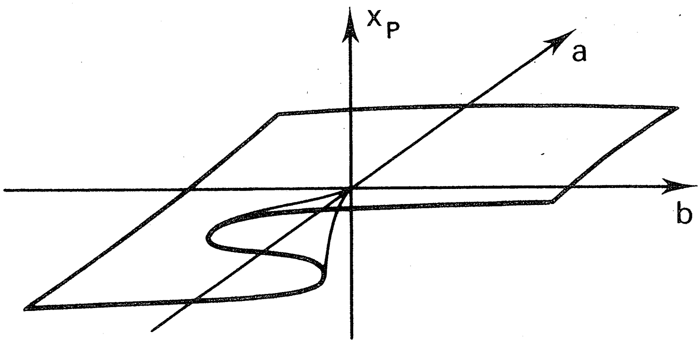


Figure 8

this figure which has become almost the trademark of Catastrophe Theory. (Although whether it is capable of supporting all the alleged applications being hung on it - from behaviour of dogs to stock exchange crashes, from fashions in curricula to optical caustics, from prison riots to falling in love - is another, and moot, point.)

The two different behaviours of the quartic function are separated by the curve

$$8a^3 + 27b^2 = 0,$$

† See Problem 2.1.

shown in Figure 9. When $8a^3 + 27b^2 < 0$, two minima exist (i.e. for values of (a, b) lying under the curve); in the case where the inequality is reversed, there is a single minimum. The point on the curve of Figure 9 is called a cusp and hence this catastrophe is termed the *cusp catastrophe*.

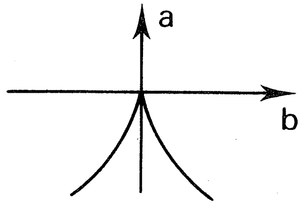


Figure 9

In our case of the boiling water, a quantity called "entropy" is maximised - or equivalently its negative, the negentropy, is minimised. The exact definitions of entropy and negentropy need not concern us here. All we need to know is that the negentropy can be locally approximated by a quartic (*), where the control variables a, b are functions of temperature and pressure.

Now, for some quartics, such as that of Figure 10, there are two minima, while for others (as in Figure 11), there is only one.

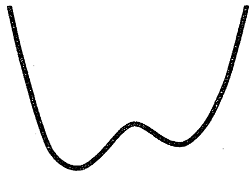


Figure 10

On the intuitive level, we might guess that where two minima are possible, these correspond to two possible states of our system (liquid and gaseous), while the unique minimum allows only one state to exist. In the case where two states are possible, *the one that is achieved is the one for which the minimum value is the less*. This rule is termed the Maxwell convention and it applies to our liquid-gas system. (Other cases, including mechanical systems like Figure 4, require somewhat different rules, which is why a direct analogy between boiling water and stretching cubics is not quite exact.)

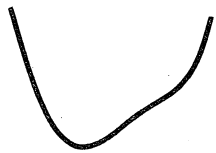


Figure 11

This allows us to make a number of predictions which are in fact borne out by experiment. First, there are critical values for p (the pressure) and for T, above which only one state can be observed and below which two are possible. Secondly, there are two ways to produce liquid-gas transitions: we might go smoothly "round the back" of the critical point P (see Figure 12) as we might come "round the front" and see a sudden change of state.

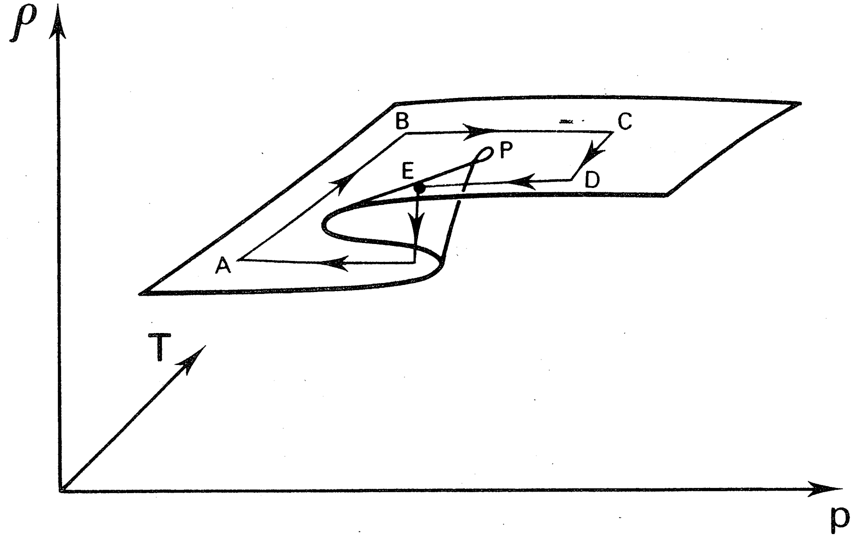


Figure 12

Figure 12 diagrams a situation in which a gas is first heated at constant pressure, then ~~expanded~~ compressed at constant temperature, then cooled back to its original temperature. This corresponds to the path *ABCD*. The point representing the state of the system has been moved from the lower surface to the upper, **i.e.** the system has changed from the gaseous to the liquid state.

The quartic negentropy function has gone through the sequence of shapes shown in Figure 13. The gas has become a liquid without appearing to liquefy in any sudden or dramatic way.

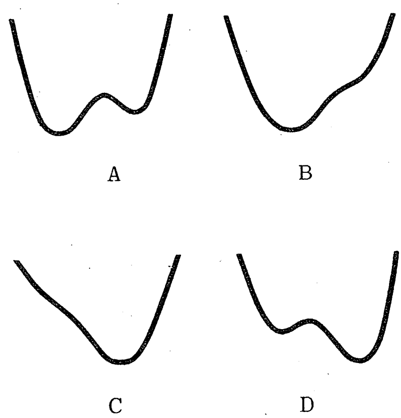


Figure 13

If we now, however, decrease the pressure, we reach a point *E* at which the liquid becomes unstable (the left-hand trough in Figure 13D increases in size until the corresponding minimum is the lower; so that Figure 13A is recovered).

There is then a sudden readjustment (boiling) and the system is once again represented by the original point A.

All this has actually been observed. There is an excellent account, in fact, in Chapter 5 of Glasstone and Lewis' *Elements of Physical Chemistry*, which goes into a lot more detail. The experiment requires fairly high pressures and is difficult (and dangerous) to do without special apparatus. Nevertheless the predictions of Catastrophe Theory correspond exactly to the experimental facts.

The mathematician who pointed out the relationship described above was D.H. Fowler, who also performed another service to the mathematical community when he translated *Structural Stability and Morphogenesis* from the original French. It was in this book that Thom stated his famous classification theorem, known as the *Theorem of the Seven.*

This result lists all the possible situations that arise for one, two, three or four control variables. There are exactly seven such cases. We have seen two already, and two more are easy to predict. These are

$$y = x^5 + ax^3 + bx^2 + cx$$

and

$$y = x^6 + ax^4 + bx^3 + cx^2 + kx$$

These have one state variable and three and four control variables respectively.

The other cases involve two state variables and are more complicated. Remarkably, there are no cases at all involving more than two state variables.

A complete list of the seven catastrophes is given in the table opposite. This lists the state and the control variables for each catastrophe and then gives the function of these that is to be minimised (or maximised). The names that follow are used as reference labels. Finally, some of the applications or hoped-for applications are listed.

The Theorem of the Seven was not fully proved by Thom. In fact the first complete proof was published as recently as 1974. More recently still, extensions have been proved for up to 16 control variables. In the main, however, the further cases that emerge are less important for applications.

From this, it will be seen that Catastrophe Theory is a very new branch of mathematics. It is a very exciting one, with wide-ranging applications and great intrinsic mathematical interest. If pursued in its full detail, it is still a difficult subject. However, a number of good expositions have appeared. If you want to learn more of this subject, you could begin with Christopher Zeeman's article *Catastrophe Theory* (Scientific American, April 1976), Ian Stewart's *The Seven Elementary Catastrophes* (New Scientist, 20 November 1976) or pages 277-285 of Stewart's Pelican book *Concepts of Modern Mathematics*. Stewart is currently writing a Pelican book on Catastrophe

State Variable(s)	Control Variable(s)	Function to be Minimised	Name	Application
x	a	$x^3 + ax$	Fold	All simple threshol phenomena.
x	a, b	$x^4 + ax^2 + bx$	Cusp	Very many; see text
x	a, b, c	$x^5 + ax^3 + bx^2 + cx$	Swallowtail	Embryology; the psychology of drunk driving.
x	a, b, c, k	$x^6 + ax^4 + bx^3 + cx^2 + kx$	Butterfly	Treatment of *Anorex Nervosa*; internatio disputes.
x_1, x_2	a, b, c	$x_1^3 - 3x_1x_2^2 + a(x_1^2 + x_1^2) + bx_1 + cx_2$	Elliptic Umbilic	Speculative applica in hydrodynamics an embryology.
x_1, x_2	a, b, c	$x_1^3 + x_2^3 + ax_1x_2 + bx_1 + cx_2$	Hyperbolic Umbilic	Buckling of stiffen panels.
x_1, x_2	a, b, c, k	$x_1^2x_2 + x_2^4 + ax_1^2 + bx_2^2 + cx_1 + kx_2$	Parabolic Umbilic	Speculative applica in linguistics and embryology.

GAMES AND MATHEMATICS

Most of us have played NOUGHTS AND CROSSES often enough to realise that the centre position is a very special one, and that if the first to play puts his mark in that position he cannot lose unless he makes a very foolish move. This means that the player who makes the first move in a game of NOUGHTS AND CROSSES has a great advantage. However he cannot generally force a win and so more often than not the game ends in a draw.

Fortunately not all games are like this. More importantly, there are many two person games which have simple rules and which are quick to play (so they make good time-wasters) but which do not have obvious strategies and which do not fade frequently into boring draws. In this article and in later issues of *Function* a number of such games will be described. All the games we choose will have simple rules, but not all the games will have simple strategies. In many games it will be difficult to determine whether or not one player has an advantage over the other.

A description of a winning strategy for one of the players in a game will involve some sort of analysis of the game. Often such an analysis is just part of a more general piece of mathematics and a recognition of this may have beneficial side effects. For it can add a bit of 'life' to what might seem a dry piece of mathematics and encourage one to look deeper into the mathematics. This in turn might lead to solutions to related but more complicated problems. We shall see an excellent illustration of this last point when, in this issue and the next, we look closely at a game called SLITHER.

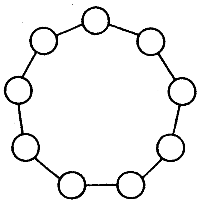
If you feel you need more motivation for thinking about puzzles than perhaps just the satisfaction of working out a strategy for a game and understanding why it works then the following quotation from George Polya may tempt you to try your hand. Polya, who is well known both as a mathematician and as the author of several books about ways of methodically tackling problems, writes in *Mathematical Discovery* Volume 1:

> "Solving problems is a practical art, like swimming, or playing the piano: you can learn it only by imitation and practice ... Our knowledge about any subject consists of *information* and *know-how* ... and in mathematics know-how is much more important than mere possession of information ... What is know-how in mathematics? The ability to solve problems – not merely routine problems but problems requiring some degree of independence, judgement, originality, creativity."

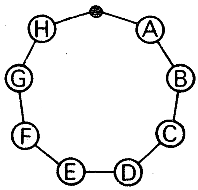
Finally, before going on to present a couple of games for your amusement, a request. As you will read elsewhere in this magazine we want contributions from our readers. If you know of a game that could be discussed here please write and tell us about it (whether or not you know how to solve it yourself).

Nonagon

A two cent coin is placed on each corner of a regular nonagon.



Two players then take turns removing either one coin or two coins from adjacent corners of the nonagon. (For example if, in his first move, the first player removes the top coin leaving



then the second player in his first move *cannot* remove, say, the pair (A, H) or the pair (A, C) of coins. (A, B) is allowed as is also (B, C).)

The player who picks up the last coin *wins*.

This game isn't fair because the second player can always win. Can you describe a winning strategy for the second player? (A solution is outlined on page 29 of this issue.)

Suppose the game were played on a regular polygon with an even number of corners. Would this make any difference?

Slither

In the June 1972 issue of *Scientific American* Martin Gardner described the following game called SLITHER. This is a game for two players played on a 5 × 6 point lattice (see Figure 1). The rules are simple. Opponents take turns marking a horizontal or vertical segment of unit length. (For example, the move shown in Figure 2 is permitted but the move shown in Figure 3 is not.)

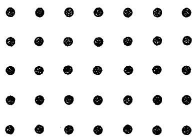


Figure 1

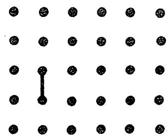


Figure 2

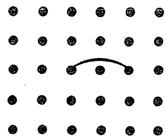


Figure 3

The segments must form a continuous path but at each move a player may add to either end of the preceding path. The player forced to close the path is the *loser*. (Figure 4 shows a position in which the next play must be a losing one.)

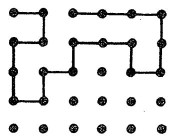


Figure 4

At the time Gardner wrote his article no winning strategy for either player was known. In a tabulation of several hundred games the wins were about equally divided between the first and the second player so there was no indication of whether one player has an advantage. Soon after publication of his article Gardner received a flood of correspondence containing strategies of steadily mounting generality until finally Ronald C. Read, a graph theorist at the University of Waterloo in Ontario, Canada, reduced the standard game to a monumental triviality.

Try to find a winning strategy for one of the players in the game of SLITHER just described. (One solution is given on page 30 of this issue). You might also try to formulate a general strategy for the game played on a 'field' of $m \times n$ dots for any numbers m and n. (A solution will be discussed in a later issue of *Function*.) Finally perhaps think about different shaped 'fields' on which the game might be played.

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That flower of modern mathematical thought – the notion of a function.

Thomas J. McCormack, 1899

CYCLONES AND BATHTUBS
WHICH WAY DO THINGS SWIRL ?

by K. G. Smith, University of Queensland

A perennial discussion point at parties is "Which way does the water swirl as it goes down the plug-holes of a bath?" In many instances the discussion becomes more or less heated, until someone suggests an experiment, and the group adjourns to the bathroom. The appropriate experiment is performed, and half the group have their claim verified. The other half of the group insist on a repetition of the experiment, and, to the surprise of the first half, and the delight of the second half, the direction of swirl is reversed. A series of experiments follow, using both the hot and cold taps, and all are mystified when no systematic pattern is found: the direction of swirl appears to vary in a random fashion.

At this point the group probably returns to the bar, and continues discussion of some other topic, such as the weather. Someone mentions that winds blow clockwise around the centre of a cyclone. A recent arrival from the U.S.A. takes exception to this statement and claims that winds blow anticlockwise around hurricanes,using his terminology. This argument is quickly settled by referring to the newspaper: the weather map shows winds blowing clockwise around low-pressure centres (cyclones) and anticlockwise around high-pressure centres (anticyclones). The U.S.A. visitor is unconvinced; he is sure his memory is not at fault. Then someone remembers reading that winds around cyclones blow in opposite directions in the northern and southern hemispheres. Honour is satisfied, and the party proceeds peacefully.

Mathematicians at the party may feel somewhat uneasy, however. The force on the air near a cyclone is towards the low pressure area in the centre. With a vague recollection of Newton's laws of motion, there arises the awkward thought: "The winds should blow towards the low pressure area, not around it". The following notes provide a brief explanation of the behaviour of bathtubs and cyclones, and will enable you to pose as an expert at any future party arguments.

Newton's Laws of Motion

In 1687, Isaac Newton published his most famous work "The Mathematical Principles of Natural Philosophy"†. In it he enunciated the laws governing motion of bodies. Only the first two are needed here; translated into modern terminology

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† *Philosophiae Naturalis Principia Mathematica,* briefly the *Principia*.

they may be given as follows:

(i) Any body will remain in a state of rest, or uniform motion in a straight line, unless acted on by an external force.

(ii) The rate of change of velocity is proportional to, and in the same direction as the applied force.

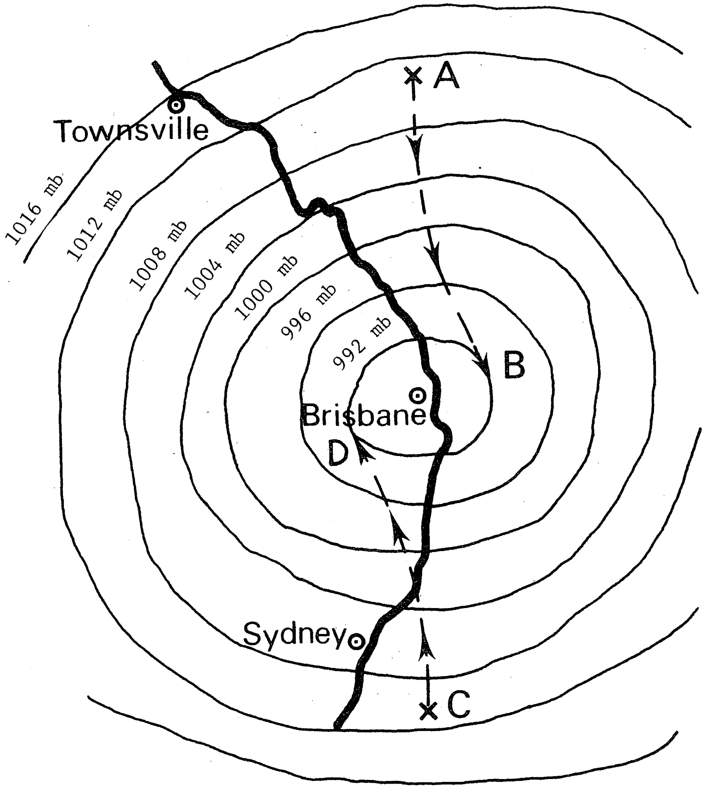
These laws are quite straightforward but have some subtleties associated with them. The significant one here is "straight line": straight line relative to what? If I am sitting in a car travelling along an unsurfaced road, and, with the aid of a straight-edge, draw a straight line on a piece of paper, the point of the pencil will trace out a straight line relative to the paper. It is unlikely to trace out a straight line relative to the car, and its path relative to the surface of the earth will be nothing like straight.

For many applications sufficient accuracy may be attained by considering motion relative to the earth. However, there are limitations: the surface of the earth is curved, and it may be suspected that if motion over large distances is considered, some corrections are needed. Generally speaking, motion over large distances will involve large time intervals, and a further complication is possible. The earth is rotating, even though very slowly, and after a large elapsed time things are in a different place. How do we modify the laws of motion to cope with these problems? Newton's laws of motion are valid provided the frame of reference is taken to be fixed (or moving at a uniform speed and not rotating) with respect to the distant parts of the universe. It is possible to derive the modifications needed to treat motion relative to a frame of reference fixed to the surface of the earth. This frame of reference will rotate slowly (once in 24 hours) and will have various small accelerations due to the motion of the earth through space. It is even possible to use a frame of reference fixed to a car moving along a bumpy road, though this becomes extremely complicated. The modifications to Newton's laws, however, are all somewhat complicated, and it is easier to consider ourselves viewing the rotating earth from somewhere in space, and using Newton's laws as he gave them.

Cyclones

A complete description of the motion of air around cyclones involves some quite advanced mathematical techniques. It is, however, possible to give a simple explanation of the origin of the clockwise airflow; all that is needed is Newton's first two laws of motion.

Consider a stationary low pressure area over Brisbane, as shown in the picture. Let us assume that initially the air is not moving relative to the earth. Then due to the pressure difference a particle of air at point A, directly north of Brisbane, will start moving south. Now the force



Paths of Particles of Air

is directly south. However, from our viewpoint in space, both Brisbane and point A are moving east, due to the rotation of the earth. The essential part of the explanation is that *since* A *is further from the axis of the earth than Brisbane is* (since A is closer to the equator), *then* A *is moving eastwards faster than Brisbane*. Thus, to look at matters very simply, by the time the particle reaches point B at the latitude of Brisbane, it will have moved further east than Brisbane; it will not continue in the direction of the dotted path since pressure forces will oppose it; more complicated reasoning shows that it will move around Brisbane. Similarly, a particle of air at C due south of Brisbane will have a smaller easterly speed than Brisbane. Thus, by the time it reaches D at the latitude of Brisbane it will not have moved as far to the east, or, relative to Brisbane, it will have moved west. The reason for the clockwise motion of air around the cyclone is now obvious. It can now be seen why cyclones do not arise in the vicinity of the equator:

all points have virtually the same easterly speed. It is clear now why air swirls anticlockwise around cyclones in the northern hemisphere.

Bathtubs and naval battles

The flow of water out of a bathtub can be treated by analogy with cyclones; the plug hole takes the place of the low pressure area, since water tends to flow towards it. We therefore conclude that water *should* swirl clockwise around the plug hole; why then do experiments give unpredictable results?

The answer lies in the magnitude of the effect we are looking at. The surface of the water is horizontal. so we need the component of the earth's spin about the vertical. The latitude of Brisbane is about $27\frac{1}{2}^\circ$, so that rate of rotation is (sin $27\frac{1}{2}^\circ$)th of a revolution per day, or about 1 revolution in 52 hours. Thus, if any residual motion in the bathwater is larger than about this rate it will completely swamp the effect of the earth's rotation. The disturbance caused by pulling out the plug, to say nothing of getting out of the bath, will be much larger than this.

Precise experiments were not made on this problem until 1961; full details may be obtained from the article "A note on the bathtub vortex" in the *Journal of Fluid Mechanics*, vol. 14, pages 21-24, 1961. A circular tank, 30 cm in diameter, was used, with a centrally located plug which could be removed from below. The results are best left in the author's own words; remember that these experiments were carried out in the U.S.A.

"Observations using powder and dye techniques quickly showed that for settling periods of a few hours or less (i) the direction of rotation of the vortex coincided with the direction in which the tub was filled, and (ii) the strength of the vortex decreased as the settling period increased, that is, as the residual circulation decreased. While these results were being obtained, the author learned that Shapiro using a six-foot-diameter tub and settling periods of several days had obtained a consistently counter-clockwise direction of rotation independent of the direction in which the tub was filled, a result attributable to the action of the Coriolis force in the northern hemisphere."

The conclusion to be drawn is that if you wish to demonstrate this effect to your friends you should acquire a large circular tank, with a centrally located plug which can be removed from below, and wait a week after filling the tank.

An interesting story is recorded on page 51 of the book "A Mathematician's Miscellany", by J.E. Littlewood (Methuen, 1953).

"I heard an account of the battle of the Falkland Islands

German ships were destroyed at extreme range, but it took a long time and salvos were continually falling 100 yards to the left. The effect of the rotation of the earth is similar to 'drift' and was similarly incorporated in the gun-sights. But this involved the tacit assumption that Naval battles take place around about latitude 50°N. The double difference for 50°S and extreme range is of the order of 100 yards."

By using the appropriate complicated mathematics, it can be shown that if a projectile is fired with a fairly flat trajectory, the deviation due to the rotation of the earth will be independent of the bearing of the target, and will be of magnitude $(a\ R^2/V)\ \sin L$, where a is the angular velocity of the earth ($= 2\pi/86{,}160$ radians/sec.), R is the range, V is the muzzle velocity and L is the latitude. You may like to check the figure of 100 yards, using some reasonable values for R and V.

Remarks

The above phenomena (cyclones, bathtubs and projectiles) can be explained by the rotation of a frame of reference fixed to the earth. Relative to such a frame of reference the equations of motion do not take the form expressed by Newton's second law; an additional term appears involving the angular velocity of the frame of reference. This additional term is sometimes called a "fictitious force", and is known as the Coriolis force, after the 19th century mathematician who investigated moving frames of reference in some detail. There are other fictitious forces associated with non-uniform motion of frames of reference. Two of these are familiar to all car travellers. When the brakes are applied there *appears* to be a force pushing you forward, *relative to the car*; when going round a bend there *appears* to be a force pushing you outwards *relative to the car*. The first of these is known as the inertial force, the second as the centrifugal force. They are both fictitious forces; in fact your body is trying to move in a straight line at a constant speed, as laid down by Newton's first law. It is only the non-uniform motion of the frame of reference (i.e. the car) which gives the apparent forces.

If the earth was spinning much faster, Coriolis effects would be familiar to us all. However, large Coriolis effects would bring additional problems; a quotation from Chapter 21 of "Rendezvous with Rama", by Arthur C. Clarke, is a suitable conclusion.

"So there was the origin of the sound they had heard. Descending from some hidden source in the clouds three or four kilometres away was a waterfall, and for long minutes they stared at it silently, almost unable to believe their eyes. Logic told them that on this spinning world no falling object could move in a straight line, but there was something horribly unnatural about a curving waterfall that curved sideways, to end many kilometres away from the point directly below its source. 'If Galileo had been born in this world,' said

Mercer at length, 'he'd have gone crazy working out the laws of dynamics'. 'I thought I knew them,' Calvert replied, 'and I'm going crazy anyway. Doesn't it upset you, Prof?' 'Why should it?' said Sergeant Myron. 'It's a perfectly straightforward demonstration of the Coriolis effect. I wish I could show it to some of my students.' ".

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THE TAPE RECORDER DIFFERENCE EQUATION†

by F. J. M. Salzborn and J. van der Hoek,

University of Adelaide

When recording music on a tape recorder, the tape winds off the "feeder" spool and passes through the machine over the recording heads at a constant speed v (usually given in inches per second), known as the tape speed. A counter is attached to the "feeder" spool which counts its number of revolutions. (On some tape recorders this counter may count every second or every third revolution.) One will notice that when the "feeder" spool is almost full, the counter "ticks over" slowly, but as the "feeder" spool empties, the counter "ticks over" more rapidly. An interesting problem then arises: Given a reading on the counter, how much recording time is there remaining or how much recording time have we used? In the analysis below we make the following assumptions.

(i) The tape has constant thickness τ.

(ii) The tape has been wound uniformly onto the "feeder" spool. (This situation is best attained by winding the tape through and then back at fast speed.)

Let $t(n)$ minutes be the time elapsed when the counter reads n. We assume that $t(0) = 0$, that is, we set the counter initially at 0000. Now make your own set of data and tabulate as shown opposite. $T(n)$ is the recording time left when the counter reads n. $T(n)$ is given by the formula

$$T(n) = E - t(n)$$

where E is the total time for running the tape at the recording speed v. You should now plot $t(n)$ against n,

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† Reproduced from Trigon, Volume 14, Number 2, July 1976. Trigon is the school mathematics journal of the Mathematical Association of South Australia. For information write to the Editor, Trigon, Department of Mathematics, University of Adelaide, Adelaide, 5001.

n	$t(n)$	$T(n)$
0	0	T
20	.	.
40	.	.
.	.	.
.	.	.
.	.	.

as in Figure 1 and Figure 2. As the tape winds off the "feeder" spool the radius of the tape left on the spool decreases. Let $r(n)$ be this radius when the counter reads n. Then $r(0) = R$, the radius of the full tape spool. Using assumptions (i) and (ii) we conclude, remembering that τ is the thickness of the tape, that

$$r(n) = R - n\tau \qquad \ldots (1)$$

and as a good approximation,

$$v(t(n+1) - t(n)) = 2\pi r(n) \qquad \ldots (2)$$

that is, the length of the tape which passes over the heads during the $(n + 1)$th rotation (in time $t(n + 1) - t(n)$) of the feeder spool, equals the circumference of the feeder spool when its radius is $r(n)$. Hence

$$t(n+1) - t(n) = \frac{2\pi}{v} r(n) = \frac{2\pi}{v}(R - n\tau).$$

Now let $\alpha = \pi\tau/v$ and $\beta = 2\pi R/v$; then

$$t(n+1) - t(n) = \beta - 2\alpha n. \qquad \ldots (3)$$

We wish now to solve (3) for t in terms of n. Consider the following list:

$$\begin{aligned}
t(n) - t(n-1) &= \beta - 2\alpha \cdot (n-1) \\
t(n-1) - t(n-2) &= \beta - 2\alpha \cdot (n-2) \\
t(n-2) - t(n-3) &= \beta - 2\alpha \cdot (n-3) \\
&\;\;\vdots \\
t(2) - t(1) &= \beta - 2\alpha \cdot 1 \\
t(1) - t(0) &= \beta - 2\alpha \cdot 0
\end{aligned}$$

If we add up all these terms on the left we obtain $t(n) - t(0) = t(n)$; and by adding up the right hand side we obtain

But $1 + 2 + 3 + \cdots + (n - 1)$ is an arithmetic series with common difference 1 and with sum equal to

$$\frac{n(n-1)}{2}.$$

Hence

$$\begin{aligned} t(n) &= n\beta - \alpha n(n-1) \\ &= -\alpha n^2 + (\alpha + \beta)n \\ &= -\alpha n^2 + \gamma n, \end{aligned} \qquad \ldots (4)$$

where $\gamma = \alpha + \beta$. We must now assign values to α and γ. This can be done by timing the tape at a given recording speed for say 400 and 800 revolutions on the counter. Then

$$t(400) = -160\,000\alpha + 400\gamma$$
$$t(800) = -640\,000\alpha + 800\gamma$$

Hence, solving the equations,

$$\alpha = (2t(400) - t(800))/320\,000$$
$$\gamma = (4t(400) - t(800))/800.$$

	1200' Agfa PE31 Reel to Reel tape	C90 Hitachi U/D Cassette
v	$3\frac{3}{4}$ ips	$3\frac{3}{4}$ ips
$t(400)$	25·4 minutes	23·5 minutes
$t(800)$	45·6 minutes	41·1 minutes
α	0·000 016	0·000 018
γ	0·070	0·067

You should now be able to calculate your own values for α and γ, and so obtain a formula for your own tape and tape recorder. The graphs in Figures 1 and 2 display theoretical and actual values of $t(n)$ against n. The theoretical values in each case are given by the curve; the actual values have been encircled. Note that for different tape speeds and for different brands of tapes (different thicknesses), you will need to recalculate the values of α and γ.

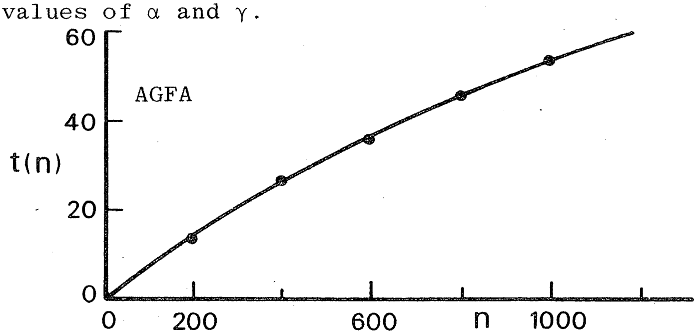

Figure 1

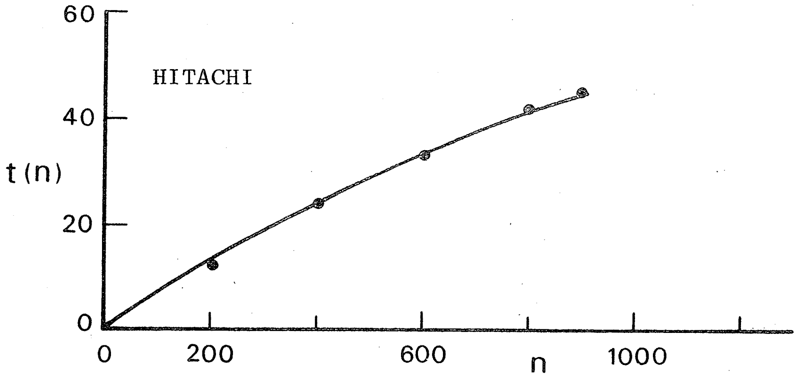

Figure 2

This graph should be kept handy when you do your recording. Of course the larger the graph you draw the more accurately you can read off your times!

Research problem: Can you work out a similar theory for relating the width of a track on a record with time needed to play it? Of course you will have to make some assumptions, for example that the grooves are equally spaced.

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PROBLEM 2.1. (Part (a) is problem B4 of the H.S.C. examination paper in Pure Mathematics in 1975.)

(a) A curve has equation $y = 3x^4 - 4x^3 - 6ax^2 + 12ax$, where a is a positive constant. For what values of x does the curve have a horizontal tangent? Determine the nature of all stationary points if (i) $0 < a < 1$, (ii) $a = 1$.

Sketch the curve when $a = 1$. State the coordinates of all stationary points but make no attempt to determine exactly the x-coordinates of any points (other than the origin) at which the curve crosses the x-axis.

(b) Extend the discussion to cover (iii) $a < 0$, (iv) $a = 0$, and (v) $a > 1$.

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THE WONDERS OF NAURU HOUSE

"The stone finish on the panels [of Nauru House, Melbourne] is small white pebbles mined from the Otway Ranges. These panels fit together in a square plan form with shaved corners which produce a hexagonal shape (four long sides and four short)."

Norman Day's Architecture article, The Age, 20 April 1977.

HOW LONG, HOW NEAR ? THE MATHEMATICS OF DISTANCE †

by Neil Cameron, Monash University

Mathematics often abstracts some concept from a range of familiar contexts, extends it beyond the confines of those contexts and in so doing gives the concept more precision and ourselves a better understanding of it.

This has, for example, happened with the metaphor of *distance* which is so much part of our thinking and language: unsuccessful generals may believe their strategies to be *close* to that of Napoleon, while a rumour may be a *long* way from the truth. We will be understood if we express the opinion that, as musicians, Mozart and Haydn are fairly close to each other while Haydn is a long way from Rod Stewart. We might even represent this by a geometrical diagram as in Figure 1 and deduce that Mozart also is a long way from Rod Stewart.

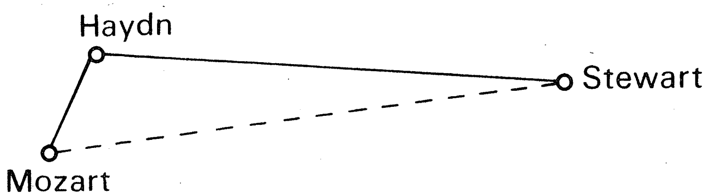
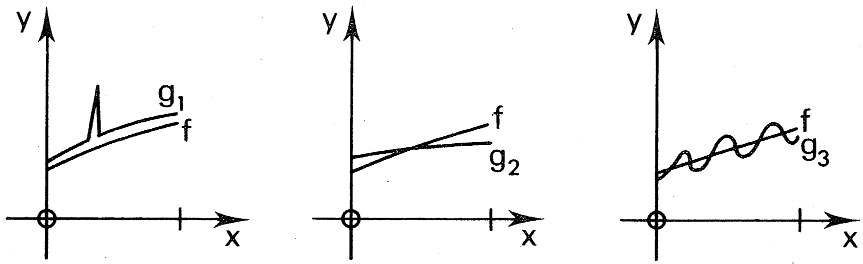


Figure 1

In mathematics, the term *metric space* has been coined for any non-empty set X of objects together with a concept of *metric* or distance $d(x,y)$ between objects x, y in X. The set X can be quite exotic, for example its objects may be vectors, matrices, functions or even operations such as integration. It might be more unusual to come across a metric space of generals or of musicians.

Given a set X, there may be many different yet sensible ways of measuring distance between objects in X. Consider continuous functions f and g as shown in the graphs. We ask if g is close to f? Is g a good approximation to f? In each of the three cases, in certain circumstances, the answer may be yes.

† This is the text of a talk to fifth and sixth formers given at Monash University on March 25, 1977.



(a) (b) (c)

Figure 2

If we measure the distance $d(f,y)$ between f and g by using the *area* between the graphs† this may be small in all cases, so that according to this distance criterion each g may approximate f suitably well. In Figure 2(a), the values of g_1 are quite different from those of f on a small section of the domain. If we wish to prepare tables, such as those correct to a specified number of decimal places, in which we guarantee that no single entry is in error by more than a stated small amount, we might use $d(f,g) = \max\{|f(x)-g(x)| : 0 \leq x \leq 1\}$, and then although the error is localised, the metric distance between f and g_1 may be too large to be acceptable. By this test g_2 and g_3 may be satisfactory as approximations to f. For some purposes, g_3 ought not to be regarded as close to f. The graph of g_3 is much longer than that of f and the gradient patterns are very different. A metric which reflects these factors will identify g_3 as a poor approximation to f.

Once the underlying phenomenon, here distance, is recognised, the mathematician grasps the opportunity to abstract it, develop an appropriate theory and then apply the results to the varying contexts. This results, not only in a saving of effort, but also in a deeper understanding of the phenomenon.

One fundamental property of distance is expressed by our first diagram or perhaps better by the assertion that "a journey cannot be shortened by breaking it." Not every situation where the distance metaphor is used has such a geometrical interpretation. For example in society, it is quite possible for Brown to be very friendly with (or close to) both Smith and Jones, but for Smith and Jones to hate each other (or be very far apart).

† The area between f and g is measured by the integral $\int_0^1 |f(x)-g(x)|\,dx$.

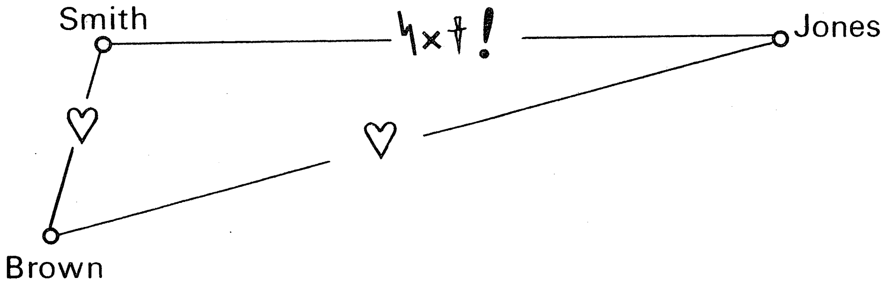


Figure 3

We cannot express this (the eternal triangle) by a diagram in which friendship is indicated by closeness; indeed this difficulty has bedevilled personal relationships and international diplomacy from the beginnings of mankind.

A distance function $d(x,y)$ defines a *metric space* if it satisfies the following conditions (the defining axioms for metric spaces):

(M1) $d(x,y)$ is a non-negative number,

(M2) $d(x,y) = 0 \Leftrightarrow x$ and y coincide,

(M3) $d(x,y) = d(y,x)$,

and in line with the earlier discussion, the so-called triangle inequality

(M4) $d(x,y) + d(y,z) \geqslant d(x,z)$.

The listed axioms are few and simple to understand yet on this foundation has been built a considerable branch of modern mathematics, whose results are useful in such varying fields as physics and economics.

Let me explore briefly only a few ideas of the theory. If $a \in X$ and $k > 0$, define the *open ball* centred at a of radius k as the set

$$B(a;k) = \{x \in X : d(x,a) < k\}.$$

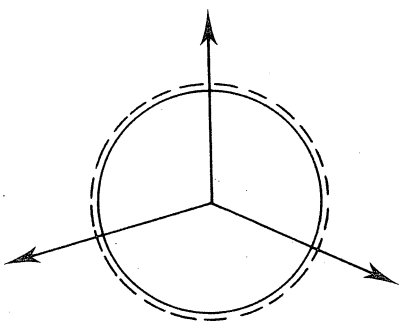


Figure 4

If X is familiar three-dimensional space with the usual euclidean concept of distance such balls really do look like balls (with outer skin removed, because of the $<$ symbol). In general their appearance can be quite different. For example let X be a cartesian plane and define the distance between two points in X as the larger of the distances between the projections of the points onto the x-axis and y-axis respectively. (This metric is sometimes called the chess-king metric as it measures the minimum distance covered by a king in moving from point to point on an infinite chess-board.) In this situation the open ball of radius 1 centred at the origin is square in appearance, as in Figure 5.

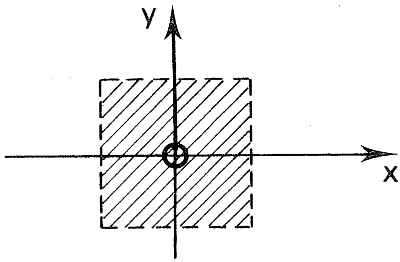


Figure 5

A situation we are familiar with in Melbourne is the radial rail network, in which the usual distance notion is radically warped. Consider a simplified situation (Figure 6) in which X consists of long spokes spaced $\pi/6$ apart, centred at Flinders Street. Travel is restricted to this rail network. If P is a point 5 km out on a spoke from Flinders Street then the open ball centred at P of radius 7 km appears as on Figure 6.

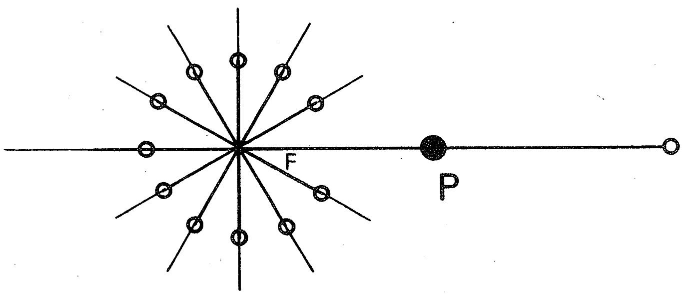


Figure 6

Let us prove that in every metric space every pair of distinct objects can be separated by disjoint open balls. This is an important result in the theory and the proof demonstrates the importance of the axioms. Let the pair be

a and b. By (M1), $d(a,b) \geqslant 0$. In fact, since $a \neq b$, by (M2) $d(a,b) > 0$. Write $d(a,b) = k$. Then $B(a\ ;\ \frac{1}{3}k)$, $B(b\ ;\ \frac{1}{3}k)$ will do.

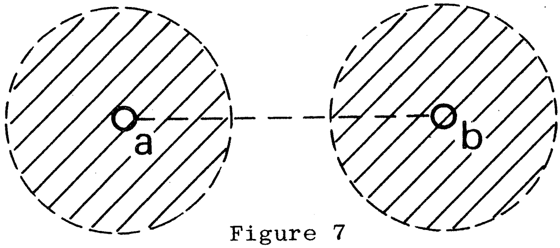


Figure 7

For, suppose these are not disjoint. Then for some x,

$$d(x,a) < \frac{1}{3}k \text{ and } d(x,b) < \frac{1}{3}k.$$

By (M3), $d(a,x) = d(x,a)$ so $d(a,x) < \frac{1}{3}k$.
By (M4), $d(a,b) \leqslant d(a,x) + d(x,b) < \frac{2}{3}k$.
So $d(a,b) < \frac{2}{3}k$. But $d(a,b) = k$, so we have a contradiction. Thus the balls are indeed disjoint.

Study this and see how vital the axioms are in the proof. For yourself, relax (M3) and note where the argument breaks down. If (M3) does not hold, redefine $B(a\ ;\ k)$ symmetrically as $\{x \in X : d(x,a) < k \ \&\ d(a,x) < k\}$ and prove that the separation result still holds in such a space. This sort of adaptation is at least worth noting since situations not modelled by metric spaces may be modelled by spaces with this weaker distance notion. Returning for a moment to the axioms for a metric space and as an exercise for yourself, show that one axiom is superfluous.

We can go on to define a subset Y of a metric space as a *neighbourhood* of an element a of Y if there is some ball $B(a\ ;\ k)$ contained in Y ; Y is an *open set* if it is a neighbourhood of *all* elements of Y. It can then be proved that

1. X and the empty set $\emptyset$ are both open sets.
2. The union of any family of open sets is itself an open set.
3. The intersection of finitely many open sets is itself an open set.

If you can prove this (it is not too difficult), you will have shown that every metric space is a *topological space* for the more abstract concept of a topological space is precisely a set X with a family of subsets, called open, which satisfy the above three axioms.

From there you can go on to a study of topological spaces in which there may be no underlying metric, but only the idea of neighbourhood expressed using the open sets.

Books

W.G. Chinn & N.E. Steenrod, 'First Concepts of Topology', New Mathematical Library (Random House/Singer).

B. Mendelson, 'Introduction to Topology', Allyn & Bacon.

The next two problems will extend your knowledge of topological spaces.

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PROBLEM 2.2.

If X is a cartesian plane and, for all points P, Q in X, $d(P, Q)$ is defined as $|x - u| + |y - v|$, where (x, y) are the coordinates of P and (u, v) those of Q, verify that d is a metric on X. Draw the open ball $B((0, 0); 1)$ in this metric space.

PROBLEM 2.3.

If X is a cartesian plane and, for all points P, Q in X, $d(P, Q)$ is defined as 0 if $P = Q$, and 1 if $P \neq Q$, verify that d is a metric on X. Describe the open balls $B((0, 0); 2)$ and $B((0, 0); \frac{1}{2})$ in this metric space. Verify that every subset of this metric space is open.

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SOLUTION TO 'NONAGON'

The winning strategy for the second player that we outline is based on symmetry. Suppose that in his first move the first player removes two coins leaving the board as shown in Figure 1 below. The second player should then take the coin exactly opposite so that after the second player's move the board looks like Figure 2.

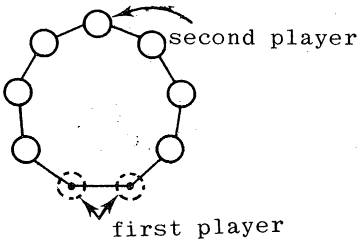

Figure 1

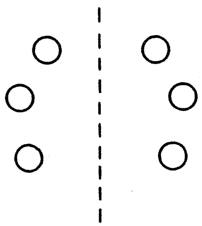

Figure 2

Notice that after these moves the coins are left in two groups symmetrical about the dotted line. From this point on all the second player has to do is to make moves which "preserve" the symmetry of the remaining arrangement about the dotted line. (You can make a table of all the possible moves from this point on and check that by playing as instructed player 2 will win.)

On the other hand if the first player removes only *one* coin in his first move, can you describe what the second player's first and later moves should be in order to be sure that he will win?

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SOLUTION TO 'SLITHER' ON A 5 X 6 RECTANGULAR FIELD

On such a field the first player has an easy win by taking the central edge and thereafter making his moves symmetrically opposite to his opponent's moves.

In the next issue we shall describe solutions to more complicated versions of SLITHER. You are invited to submit your ideas about generalizations of the game (and their solutions) to the Editors.

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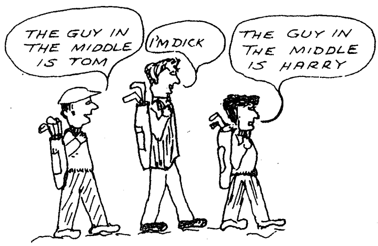
PROBLEM 2.4.



Three golfers named Tom, Dick, and Harry are walking to the clubhouse. Tom, the best golfer of the three always tells the truth. Dick sometimes tells the truth, while Harry, the worst golfer, never does.

Figure out who is who.

(Hint: First figure out which one is Tom.)

PROBLEM 2.5. (This is Problem 1.4, modified. See solution below.)

(i) The right hand digit of a natural number is to be removed and replaced at the left hand end, so increasing the original number by fifty per cent. Prove that this is impossible.

(ii) Repeat part (i) with fifty replaced by seventy five. Find all solutions. (Solutions invited.)

PROBLEM 2.6.

A person A is told the product xy and a person B is told the sum $x + y$ of two integers x, y, where $2 \leqslant x, y \leqslant 200$. A knows that B knows the sum, and B knows that A knows the product. The following dialogue develops:

A: I do not know $\{x, y\}$.
B: I could have told you so!
A: Now I know $\{x, y\}$.
B: So do I.

What is $\{x, y\}$? (Solutions invited.)

PROBLEM 2.7.

A very good approximate method of calculating sin x for x between 0 and $\pi/2$ is by means of the formula

$$\sin x \approx x[1 - 0{\cdot}16605\, x^2 + 0{\cdot}00761\, x^4]$$

Use a calculator or a computer to make your own table of sin x, and compare it with published tables.

SOLUTION TO PROBLEM 1.4.

Write the number using the digits
$x = a_n a_{n-1} \cdots a_2 a_1 a_0 = a_n 10^n + a_{n-1}10^{n-1} + \cdots + a_2 10^2 + a_1 10 + a_0$.
Removing the left digit and placing it at the right yields the new number

$$a_{n-1}10^n + a_{n-2}10^{n-1} + \ldots + a_2 10^3 + a_1 10^2 + a_0 10 + a_n.$$

For this to be fifty per cent greater than the original number we require

$$2(a_{n-1}10^n + a_{n-2}10^{n-1} + \cdots + a_1 10^2 + a_0 10 + a_n) = 3(a_n 10^n + a_{n-1}10^{n-1} + \cdots + a_1 10 + a_0)$$

that is, $17(10^{n-1}a_{n-1} + 10^{n-2}a_{n-2} + \cdots + 10a_1 + a_0) = (3{\cdot}10^n - 2)a_n$.

Hence 17 divides $3{\cdot}10^n - 2$ (it can't divide the digit a_n). The lowest n for which this is true is $n = 15$, when

$$10^{14}a_{14} + 10^{13}a_{13} + \cdots + 10a_1 + a_0 = 176\ 470\ 588\ 235\ 294\ a_{15}.$$

Taking $a_{15} = 1$ (2, 3, 4 or 5 could also be chosen) we get

$$x = 1\ 176\ 470\ 588\ 235\ 294.$$

Another possible n is $n = 30$, and of course still higher values are possible.

(Solutions were received from Mark Michell and Ms E. O'Gallagher.)

I was at the Mathematical School, where the Master taught his Pupils after a Method scarce imaginable to us in *Europe*. The Proposition and Demonstration were fairly written on a thin Wafer, with Ink composed of a Cephalick Tincture. This the Student was to swallow upon a fasting Stomack, and for three Days following eat nothing but Bread and Water. As the Wafer digested, the Tincture mounted to his Brain, bearing the Proposition along with it.

Jonathan Swift: *Gulliver's Travels: A Voyage to Laputa*, 1726

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Three hundred men are arranged in 30 rows and 10 columns. The tallest man is chosen from each row and then the shortest man is chosen from these 30 men. On another occasion, the shortest man is chosen from each column and then the tallest man is chosen from these 10 men. Who is the taller, the tallest of the short men or the shortest of the tall men?

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I had a feeling once about mathematics, that I saw it all - Depth beyond depth was revealed to me - the Byss and the Abyss ... I saw ... a quantity passing through infinity and changing its sign from plus to minus. I saw exactly how it happened and why the tergiversation was inevitable: and how one step involved all the others ... .

Winston S. Churchill: *My Early Life*

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MATHEMATICS LECTURES

The series of lectures for 5th and 6th form students at Monash University continues. They are on Friday evenings, from 7 p.m. to 8 p.m. Your school has received a detailed programme.

Lectures are held in the Rotunda Lecture Theatre R1 (enquire at the main gate). The remaining lectures are:

May 27	The Foucault Pendulum, with demonstration.	Dr C.F. Moppert
June 10	Mathematics of Winds and Currents.	Dr C.B. Fandry
June 24	Number Theory.	Dr R.T. Worley
July 8	Stonehenge and Ancient Egypt; the mathematics of radiocarbon.	Dr R.M. Clark
July 22	Computing Orbits.	Dr J.O. Murphy
August 5	How Things Begin; the development of some mathematical concepts.	Professor J.N. Crossley

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ROYAL METEOROLOGICAL SOCIETY LECTURE FOR SCHOOLS

The Society announces its third annual sixth form lecture, to be held this year on Friday 24 June in the Fritz-Loewe Lecture Theatre, University of Melbourne at 8.00 p.m. The speaker will be Professor P. Schwerdtfeger from Flinders University, South Australia. The title of his talk will be

"What's the use of meteorology"

The lecture is aimed primarily towards senior students of physics, mathematics, geography and related disciplines, and their teachers. The lecture will last for about one hour with time following for questions and discussion. Supper will be served afterwards without charge.

Tickets (which are free) and further details may be obtained from Dr R.K. Smith, Department of Mathematics, Monash University, telephone 541-2556, 541-2595 or, after hours, 754-5492.

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