Mathemagical Ambigrams

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Abstract

Ambigrams are calligraphic designs that have several interpretations as written words. In this concise introduction to ambigrams we focus on the mathematical aspects of ambigrams and ambigram design.

1. Introduction

This essay has been written with a special audience in mind. You belong to this audience if you are, just like me and many other mathematicians, fascinated by Escher's drawings, puzzles, wordplay, and illusions. If you belong to this category, then it is almost certain that you will also like ambigrams. Furthermore, you also will not need to be told why it is that ambigrams are fun, you almost certainly would appreciate to see your name and those of other persons and things close to your heart ambigrammed, and may even want to know how to construct ambigrams yourself.

What we will concentrate on in the following is to summarise some basic information about ambigrams that will appeal especially to minds that are wired in a mathematical way. We do this by playing some typical games that mathematicians like to play. For example, for many of us mathematics is the study of symmetry in one form or another. Therefore, a mathematically minded ambigrammist will want to see everybody's favourite mathematical terms turned into beautiful calligraphic designs displaying unusual geometric symmetries. Or what about constructing magic squares and other mathematical puzzles that incorporate an additional ambigrammatic dimension? Or, more generally, what about fusing ambigrams with other worlds of ambiguity and wordplay, by constructing appropriate ambigrammatic captions to Escher's drawings, creating an ambigram of the word ambigram, or an ambigram of a palindrome?

2. Definition

The word *ambigram* was coined by Douglas R. Hofstadter, a computer scientist who is best known as the publizer prize winning author of the book *Gödel, Escher, Bach*. In [4] Hofstadter defines what he means by an ambigram.

> "An ambigram is a visual pun of a special kind: a calligraphic design having two or more (clear) interpretations as written words. One can voluntarily jump back and forth between the rival readings usually by shifting one's physical point of view (moving the design in some way) but sometimes by simply altering one's perceptual bias towards a design (clicking an internal mental switch, so to speak). Sometimes the readings will say identical things, sometimes they will say different things."

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The word ambigram contains the Latin prefix “ambi–” suggesting “ambiguity” and the Greek stem “–gram” which means “written specimen”. Other words that have been used to denote ambigrams include “Inversions”, “Designatures”, “Backwords”, and “Symmetricks”, to name just a few.

3. First Encounter

There are many different types of ambigrams. The most common type consists of the so-called *half-turn ambigrams*. Half-turn ambigrams have two different readings and to switch from one to the other you simply have to rotate the ambigram 180 degrees in the plane it is living in. As a first example consider the half-turn ambigram of the word *algebra*; see Figure 1. This and most of the other ambigrams we will come across in the following incorporate two identical readings.

Figure 1. A half-turn ambigram of the word *algebra.*

Other types of ambigrams include the *wall reflection ambigrams* (switch by reflecting through a vertical line in the plane), *lake reflection ambigrams* (switch by reflecting through a horizontal line in the plane) and *quarter turn ambigrams* (switch by rotating 90 degrees in the clockwise or anti-clockwise direction). Here is an example of a wall refection ambigram of the word *geometry.*

Figure 2. A wall reflection ambigram of the word *geometry.*

Wait! Before you get too excited about ambigrams, decide to give up your career and dedicate the rest of your life to the study of ambigrams, I have to warn you about something (I do the same with students asking me about a career in pure mathematics). You have to realize that, just as a mathematician or artist not engaging in an activity having any apparent commercial value, you will often be asked the dreaded question: “What is it all good for?” Since many people will not be able to understand the true reasons, you have to be ready to answer this question in a way that will satisfy even simple minds. Here is a scenario that you can use for this purpose.

Hurrying somewhere, a stereotypical mathematician approaches a glass door, absentmindedly reads a “Push” sign, and a moment later crashes into an unyielding door. Only to realize then that he made the mistake of subconsciously deciphering a halftransparent sign that was printed on the other side of the door whose message was meant for people approaching from that side. By replacing the sign by a wall reflection ambigram pair *Push/Pull*, we may be able to save innumerable glass doors and lives; see [12] for an example. Similarly, many ambulances have the word *ambulance* written in mirror writing somewhere in the front to the car so that people can easily decipher the word in their rear mirrors when the ambulance is approaching from behind. So, obviously, what is needed here is a wall reflection ambigram of the word *ambulance*.

4. Natural Ambigrams

Written in a suitable font the capital letters B, C, D, E, H, I, K, O, X have a horizontal symmetry axis. This means that all words that can be written using only these letters are *natural* lake reflection ambigrams. Examples with a nice self-referential ring to them include the words CODEBOOK, DECIDE, and ECHO.

Figure 3. MATH, CODE, and OHIO are examples of natural wall reflection, lake reflection, and quarter-turn ambigrams, respectively.

Similarly, again written in a suitable font, the capital letters A, H, I, M, O, U, T, V, W, X, Y have a vertical symmetry axis. By writing words that only contain these letters from top to bottom, we arrive at many examples of natural wall reflection ambigrams such as AIM, YOYO, MATH, and MAXIMUM. Natural half-turn ambigrams include the words NOON and SWIMS. Also NO/ON and MOM/WOW are examples of natural half-turn ambigram pairs. The word OHO is an example of an ambigram that is both of the wall and lake reflection types. Examples of natural quarter-turn ambigrams are the words NZ (abbreviation for New Zealand) and OHIO (think of the bars at the top and bottom of the letter I as being extended slightly to make it look like the H turned 90 degrees). In order to switch to the second reading you have to turn both words in the anti-clockwise direction and then read from top to bottom.

5. History

Of course, people must have been aware of these and similar natural ambigrams ever since the invention of writing. However, to the best of our knowledge, it was not until the beginning of the twentieth century that some playful minds started constructing natural ambigrammatic sentences such as the swimming-pool sign NOW NO SWIMS ON MON. Also, around that time the first non-trivial ambigrams started to appear. The earliest example that I am aware of is a half-turn ambigram pair *puzzle2/the end* in Peter Nevell's book *Topsys and Turvys–Number 2*; see [11]. In [1] Martin Gardner reproduced ambigrams of the words *chump*, *honey,* and the signature of *W.H. Hill* that originally appeared 1908 in *The Strand*.

Traditionally, logo designers have been very fond of logos consisting of a few letters that have an ambigrammatic flavour. Therefore it is surprising that it was only relatively late in the twentieth century that a number of artists independently from each other discovered/developed the fundamentals of ambigram design and started producing very professional ambigrams.

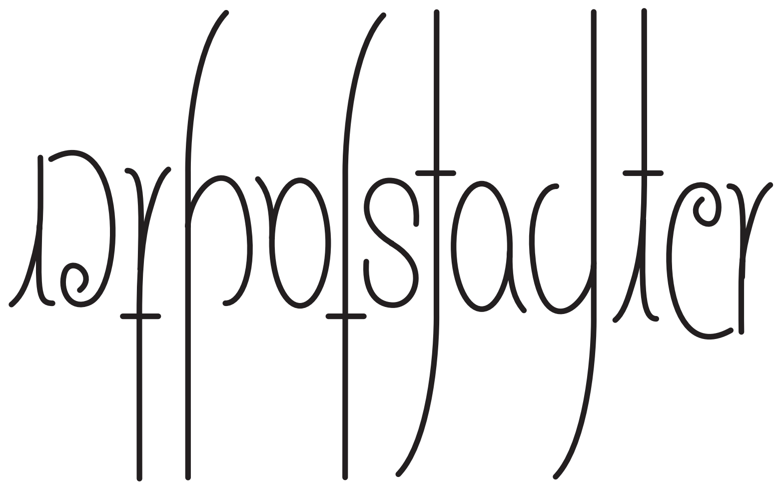

Figure 4. A half-turn ambigram of *Dr Hofstadter*. Note that the *Dr* is supposed to stand for both the academic title and the initials of Douglas R. Hofstadter.

The three artists who have contributed most to the art of ambigrams are Douglas R. Hofstadter, Scott Kim, and John Langdon who are also the authors of three books dedicated exclusively to ambigrams; see [4], [6], and [8]. All three had experimented with ambigrammatic ideas early on in their lives, but things really only started taking off around 1980 when a number of ambigrams appeared in Scott Morris's column in the *Omni* magazine and Martin Gardner's mathematical column in the *Scientific American*. Also Scott Kim's beautiful and comprehensive book on the design of ambigrams [6] has been the ambigrammist's bible since it appeared in 1981.

Douglas Hofstadter's book *Ambigrammi* was published in 1989 (in Italian). In it he introduces a number of new ingenious types of ambigrams and, using the process of ambigram design as a representative example, probes deeply into questions on how we create in general. John Langdon is a professional graphic designer and teacher of graphic design and calligraphy. In his artwork he combines graphic design, philosophy, and language into ambigrams of unsurpassed perfection.

6. Different Types

In his book *Inversions* Scott Kim outlines a rough but very sensible classification of the different types of ambigrams. He distinguishes between types that are based on *geometric symmetries*, *perceptual mechanisms*, and *other organizing principles*.

The four different types of ambigrams we have encountered so far are based on plane geometric symmetries. Just using quarter-turns around the origin and reflections through the coordinate axes, it is possible to generate symmetries on which all four and further similar types of ambigrams can be based. In fact, any kind of plane geometric symmetry can be translated into an underlying principle for making up new types of ambigrams: rotation, reflection, scaling, translation, shearing, fractals, tessellations, you name it. In fact, examples for many of these possible types of ambigrams can be found in the literature cited in the references. There are also a number of good examples based on spatial symmetries. For example, on his webpage [9] John Langdon writes the mysterious word OM using several twists of a spiral. Viewed end-on you see the letter O, and viewed from one side you see the letter M.

Figure 5. A half-turn ambigram of the word *Maths.*

While ambigrams based on geometric principles are by far the most common and popular ones, various perceptual mechanisms have also been used to very good effect. For example, both John Langdon's and Scott Kim's first ambigrams were *figure/ground* ambigrams. Similar to Escher's tilings, in a figure/ground ambigram you see one word when you concentrate on the solid part of the design and another word if you concentrate on its negative space. Very good ambigrams of this type are rare. My favourite one is a *me/you* pair by John Langdon; see again [9]. For figure/ground ambigrams of the pairs *figure/ground* and *Escher/Escher* see [6] and [8], respectively. *Containment* is another principle in this category that can be used to encode two or more different meanings in one design. Consider the following simple natural example that, on top of everything else, is a natural lake reflection ambigram. Five words are encoded in the following sequence of letters:..ODECODECODECODEC... But, of course, it is the pair CODE/DECODE that somehow jumps out and gives it a nice self-referential touch.

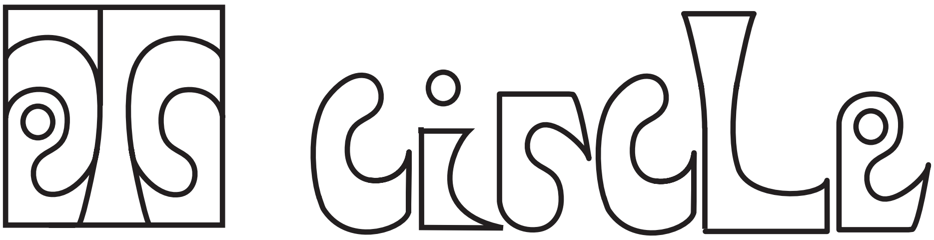

Figure 6. A dissection ambigram of *Squaring the Circle.*

Another type ambigram that does not fall into either of the categories mentioned above is the *dissection ambigram.* The example in Figure 6 illustrates that the circle can be squared after all.

7. Numbers

Why should we restrict ourselves to words when we think about ambigrams? Of course there are also ambigrams that involve numbers. By exploiting natural symmetries of Arabic and Roman numerals, it is no problem to generate any number of natural number ambigrams. Written in a suitable font the number 1961 is a half-turn ambigram, 1380 is a lake reflection ambigram, XIX is an ambigram that is both of the lake and wall reflection types, and, of course, let's not forget 8/∞, a well-known mathematical quarter-turn ambigram pair.

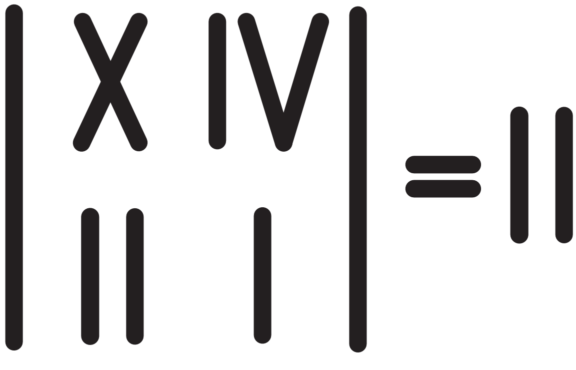

Figure 7. Half of a wall reflection ambigram pair involving 2x2 determinants.

As a first non-trivial example of a number ambigram consider the calculation involving a 2x2 determinant in Figure 7. You can find it "in action" in the Department of Mathematics at the University of Erlangen–Nürnberg in Germany. There it is printed on one of the glass doors on the first floor separating one of the corridors from the main stairway.

Look at it from one side and you verify readily that indeed

$$\mathrm{X}\cdot\mathrm{I}-\mathrm{II}\cdot\mathrm{IV}=\mathrm{II}.$$

Now move to the other side of the door and check that although we are now dealing with a different calculation it still pans out correctly:

$$\mathrm{II}=\mathrm{VI}\cdot\mathrm{II}-\mathrm{X}\cdot\mathrm{I}$$

So, what we have here is an example of a wall reflection (number) ambigram pair.

As a second completely different example consider the square array of numbers in Figure 8. After what we just said, I am almost certain that the first thing you will notice is the fact that all digits in this array turn into other proper digits after we rotate the array 180 degrees. However, the array and its rotated image are not the same.

96	11	89	68
88	69	91	16
61	86	18	99
19	98	66	81

Figure 8. Two magic squares separated by a half-turn.

What we are dealing with here is a half-turn ambigram magic square pair. All rows and columns in both squares add up to 264. I discovered this more-than-magical square in [10] and a couple of other places. None of these references mentions the name of the person who first invented it. For more examples of number ambigrams see [12].

8. Close Relatives

Ambigrams are closely related to other worlds of ambiguity and wordplay. Some important examples include ambiguous pictures, optical illusions, Escher's drawings, palindromes, and anagrams. A mathematically minded ambigrammist will almost certainly be fascinated by the idea to combine and enhance examples of those close relatives by suitable ambigrams that reinforce whatever is special about these examples. For example, many of Escher's most famous drawings have an underlying symmetry that can also form the basis for an ambigram of the title of the drawing.

The drawing *day and night* is based on a wall reflection and a tessellation. The left side of the background consists of a Dutch landscape as it appears during the day and the right side of the background features the mirror image of this scene at night. The overall impression is of the left side being white and the right side being black. However, the borderline between the two phases of the day is not sharp but is gradual as in real life. Escher works this gradual transition by letting a flock of white birds fly from the day side into the night side of his picture and a flock of black birds fly in the opposite direction. The shapes of all birds are congruent except that the birds of different colours are facing in different directions, that is, are mirror images of each other. On top of all this, the black and white birds also mesh perfectly into a tiling of the plane. To see the wall reflection ambigram pair *day/nite* in Figure 9 underneath Escher's drawing could make it even more intriguing than it already is.

Figure 9. The Escher wall reflection ambigram pair *Day/Nite*.

Many more examples of Escher ambigrams can be found in [12]. A stunning half-turn ambigram of the Escher pair *Angels&Demons* by John Langdon features prominently on the cover of a novel that was published recently; see [2]. If you are just interested in seeing the ambigram, you can view a picture of the cover at amazon.com.

Remember that a *palindrome* is a word or phrase that reads the same backward and forward. Famous examples include the words *rotor* and the phrases *able was I ere I saw Elba* (usually attributed to Napoleon which is "remarkable" since he hardly spoke any English) and *a man, a plan, a canal: Panama*! My former hometown Adelaide has the palindromic suburb *Glenelg*. Figure 10 shows a wall reflection ambigram of this palindrome. Note that in this particular ambigram every single letter has a vertical symmetry axis.

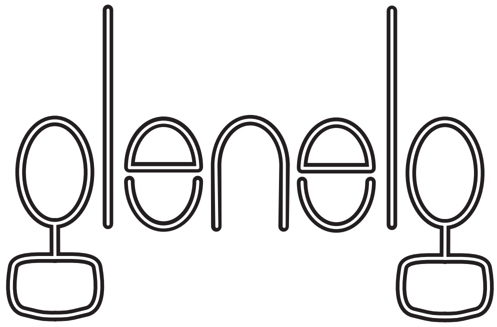

Figure 10. A wall reflection ambigram of *Glenelg,* a city in South Australia with a palindromic name.

Again, for many more examples of palindromes, anagrams, magic squares see [12]. For a very nice ambigram of the palindrome *level* see [6].

9. Do It Yourself

Ambigrams not only appeal to the mathematical mind as curious objects to behold, but also the process of making attractive ambigrams is very reminiscent of creating beautiful mathematics. In this context it is striking that both Douglas Hofstadter, and Scott Kim, two of the pioneers of ambigramming, have very strong backgrounds in mathematics. Even John Langdon, who is a professional graphic designer, is fascinated by mathematics and one of the chapters in his book is dedicated to mathematical themes.

Before we go any further, let us make up our minds what we mean by a *good* ambigram. Most ambigrammists would agree that a good ambigram should be readable, attractive, and ideally reflect some of the meaning of the word it represents. Of course, in the end it is up to the individual to decide what looks good and appealing and what does not. While a graphic designer may be mostly interested in readability and calligraphic beauty, a puzzle fan may prefer an ambigram that is a little bit of a puzzle that first needs to be solved/figured out before it can be appreciated.

I like to think of mathematics and ambigramming as rigorous art forms (of course, usually not at the same level). To be able to create and discover beauty in either of these two arenas, you have to be a master of the respective fundamental results and techniques, have a sense and taste for asking the right questions, and be very original. Finally, you also have to be determined not to settle for anything less than perfect all the way from what kind of problems you work on in the first place to how you wrap up and present your results to your target audience. We know what all this mean in the case of mathematics. What about ambigramming? To be able to give a concise answer to this question, I will pretend in the following that ambigrams are half-turn ambigrams. You should be able to generalise my remarks to whatever other type of ambigram you are interested in.

Figure 11. The letters in an ambigram only have to work in context. The symbol in the center of this design can be read both as the letter *B* and the number *3*.

The most important tool in the design of ambigrams is *the Dictionary of Half-Turn Letter Pairs.* We can extract a number of entries in this book (that only exists in the minds of ambigrammists) from our examples of half-turn ambigrams. For example, in the *algebra* ambigram we find a good *a/a* pair (actually just a self-similar *a* that turns into itself), a good *b/g* pair and a reasonable self-similar *e*. Furthermore, the *Dr Hofstadter* ambigram contains a daring *h/d* pair, good *f/t* and *o/a* pairs, and a natural self-similar *s.* In a working ambigram the good letter pairs are sometimes suggestive enough to make the word recognisable and prop up the more far-fetched part of the design. Unlike the letters of a typeface, the individual letters in an ambigram only have to work in context. The example in Figure 11 illustrates how powerful an ally context can be.

Perhaps the most simplistic approach to ambigramming is to make up a 26x26 table of symbols indexed by the letters of the alphabet in which the (*x,y*)th entry and its half-turn image are a half-turn pair for *x/y*. We can then use such a table to make up an ambigram for any word in a letter-by-letter way, by turning the first letter into the last letter, the second letter into the second last letter, and so on. In fact, there is an interactive website that does just that; see [5]. Just type in a word or even two words having the same number of letters, hit "Return", and you usually get a reasonably awful half-turn ambigram incorporating the words you started with. Despite its shortcomings this ambigram may include some good letter pairs on which a good ambigram may be based. Of course, the problem is that some letter combinations just don't admit any good fusions. Just for fun have a go at merging the letters *o* and *j.* There are some obvious ways to generalise this approach that do not work as well. For example, instead of treating individual letters as the basic chunks in a word, we could try to use two or more letter combinations as our basic chunks and then try to come up with huge tables that account for all possible chunk combinations.

What we have started doing here is to discuss ways to automate ambigramming. In his book Douglas Hofstadter provides an in-depth discussion of what is possible and what is not in terms of automated creation of good ambigrams and creation in general. One of his many conclusions is that there is no way to attack the problem in a purely combinatorial way that exhausts all possibilities.

Anyway, once you have created your Dictionary you need to learn *letter regrouping*. Letter regrouping has been used in the construction of an ambigram if different parts of one of its letters end up in different letters when you rotate the design. In our first example, the *algebra* ambigram, there is no trace of letter regrouping. On the other hand, in the *Dr Hofstadter* ambigram the *D* at the beginning of the word turns into *er* at the end of the word, and the *M* in the *Maths* ambigram turns into the letters *h* and *s*.

Figure 12. A semi-automatic way to ambigram the word ambigram.

To become very good at letter regrouping will take a bit of practice but there is a semi-automatic approach that, in conjunction with the dictionary, yields surprisingly good results. Let us ambigram the word ambigram. This is yet another self-referential game a mathematical ambigrammists will feel an urge to play sooner or later.

In step one write the word such that every one of its letters contains one, two, or three vertical strokes and such that all these vertical strokes are equally spaced all throughout the word; see Figure 12. Make an upside-down copy of the word, whip out the dictionary, and check whether there are any promising letter pairs contained in the word. In our case the *big* in the middle of the word is almost a natural ambigram. We use this cluster of promising letter pairs as an anchor when we superimpose the word and its image. By making some minor adjustments, we arrive at a quite reasonable draft of a working ambigram. Of course, as usual with examples like this you only get to see something that really works, and not all the thousands of aborted attempts.

Our draft is just a first approximation, but it already incorporates all the bits and pieces that make it work. In fact, it is basically this skeleton ambigram that is the first example in Douglas Hofstadter's book [4]. Also, purists like Hofstadter who are mainly interested in the structural skeleton of a design will usually stop at this point and not worry about turning their sketch into a professionally looking design. Note that by using our simplistic approach some letter regrouping has occurred automatically.

Figure 13. The finished half-turn ambigram of the word *ambigram*.

To turn a draft into a professionally looking design you now have to learn some of the fundamentals of calligraphic design and composition in general. You then have to study the work of others, to learn by trial and error to balance your designs and how to make the different parts of your ambigrams support each other in an optimal way. Finally, you have to get into the habit of polishing and reworking every design over and over again until even someone unfamiliar with ambigrams will be able to recognize and appreciate it.

10. Acknowledgements and Remarks

Most of what I know about ambigrams is based on the work of others, especially the books [4], [6], and [8]. This essay is only meant as a first introduction to ambigrams. For more detailed information about ambigrams and the design of ambigrams see the three books mentioned above and [12]. In addition, also check out the four websites listed in the References below.

I would like to thank Douglas Hofstadter for providing me with an English translation of his book *Ambigrammi* and John Langdon for a most encouraging and inspiring correspondence about ambigrams. Finally, thanks are due to Anu for inspiring me to get serious about creating ambigrams–my first twenty or so ambigrams were ambigrams of her name.

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[13] Burkard Polster's website: www.maths.monash.edu.au\~bpolster.